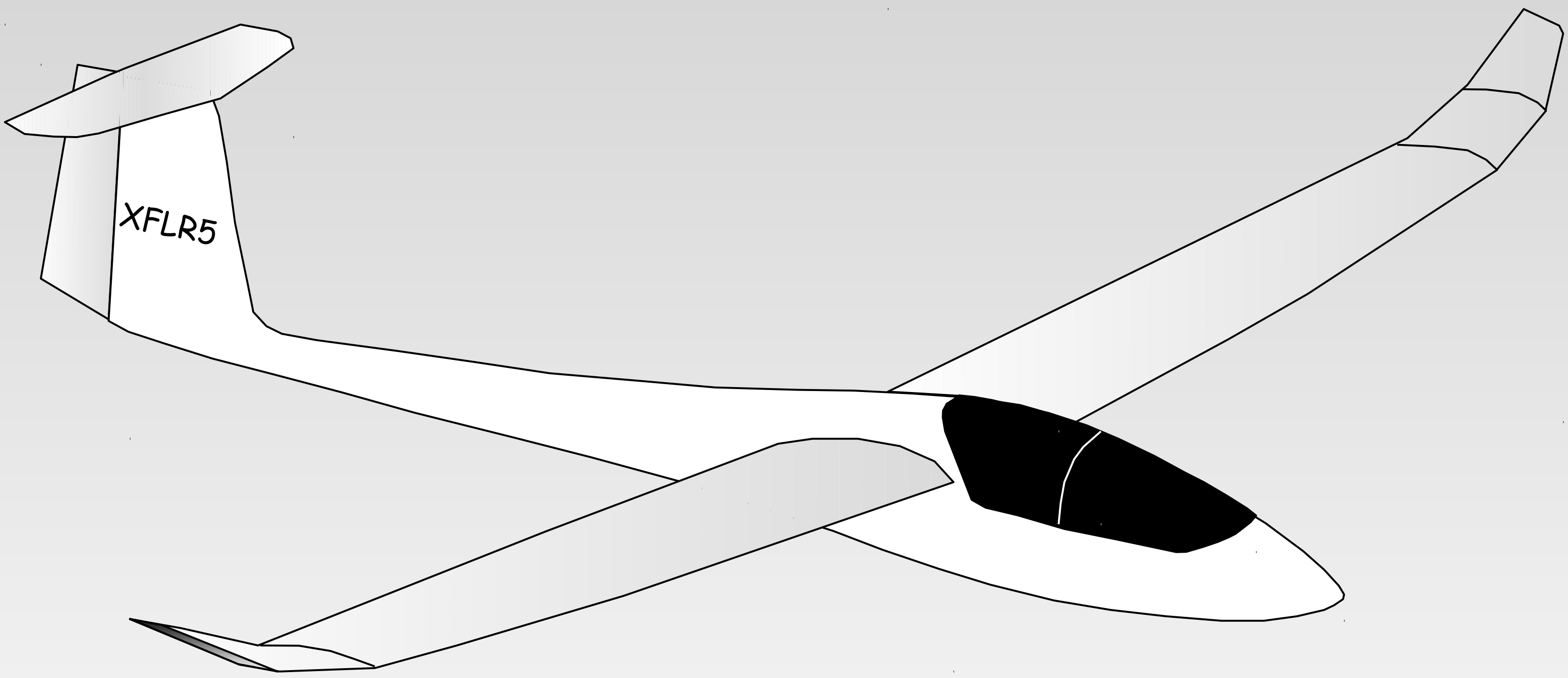
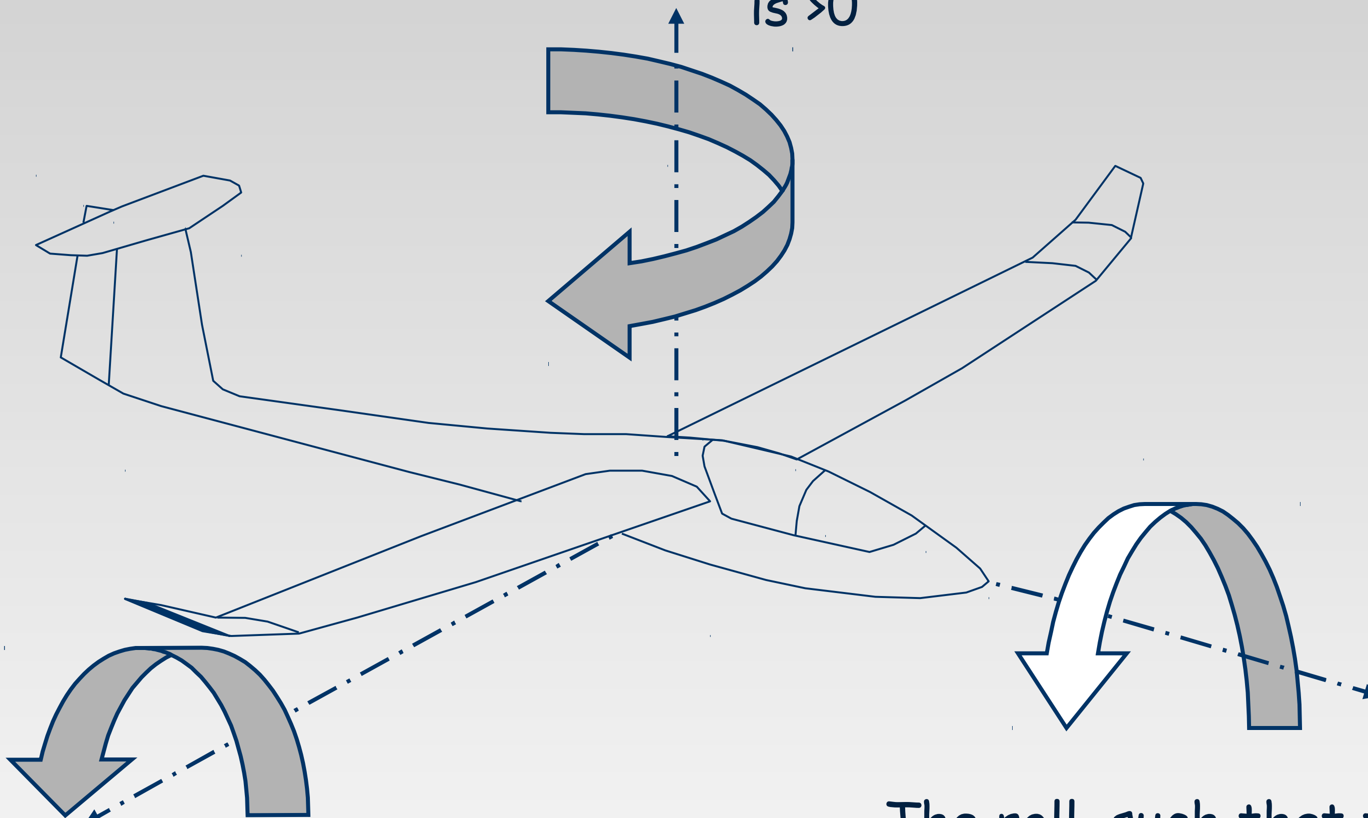
About stability analysis using XFLR5



Sign Conventions

The yaw, such that the nose goes to starboard is >0



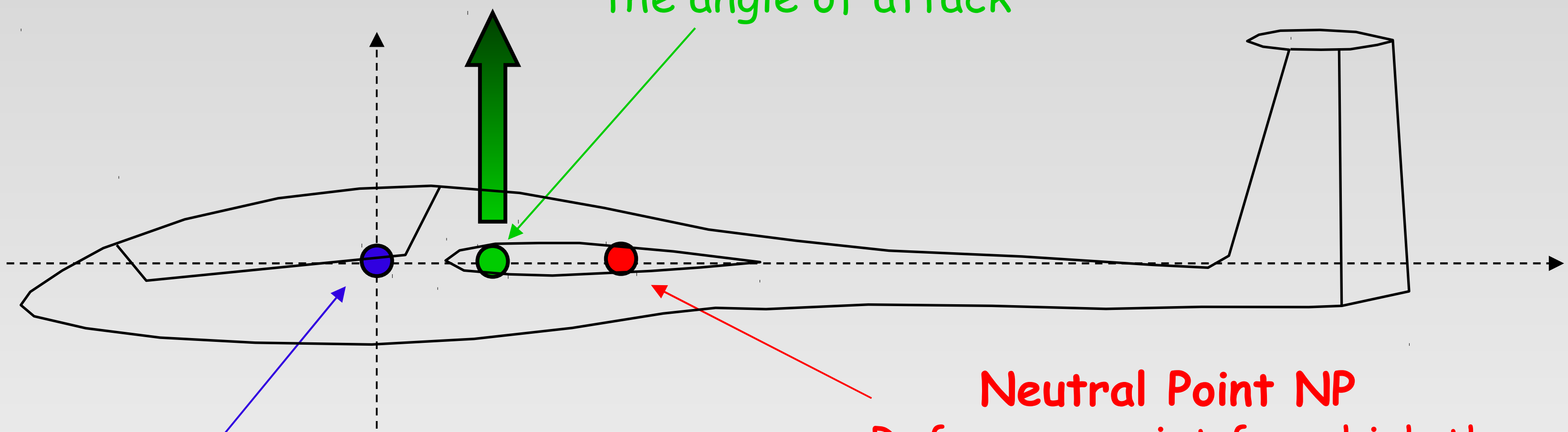
The pitching moment nose up is > 0

The roll, such that the starboard wing goes down is > 0

The three key points which must not be confused together

Centre of Pressure CP

= Point where the resulting aero force applies
Depends on the model's aerodynamics and on the angle of attack



Neutral Point NP

= Reference point for which the pitching moment does not depend on the angle of attack α

Depends only on the plane's external geometry

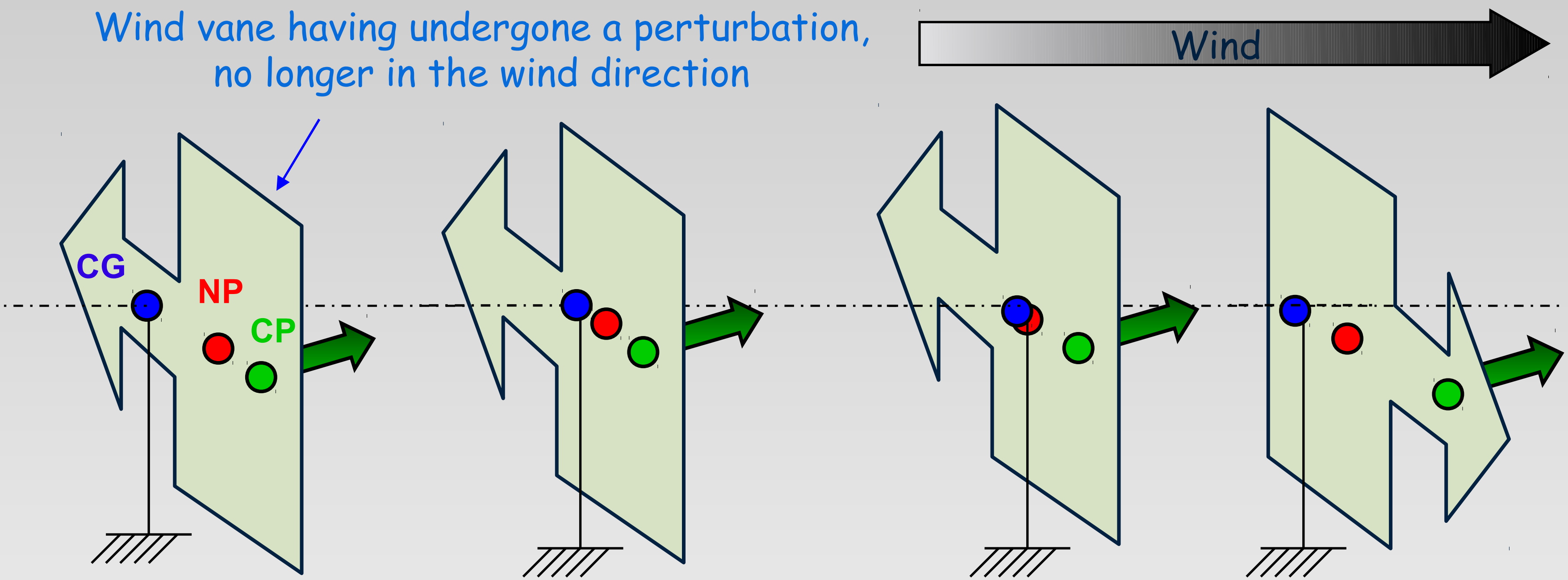
Not exactly intuitive, so let's explore the concept further

Centre of Gravity CG

= Point where the moments act;
Depends only on the plane's mass distribution, not its aerodynamics

Also named XCmRef in XFLR5, since this is the point about which the pitching moment is calculated

The neutral point = Analogy with the wind vane



CG forward of the NP
→ The pressure forces drive the vane back in the wind direction
→ Very stable wind vane

CG slightly forward of the NP
→ The pressure forces drive the vane back in the wind direction
→ The wind vane is stable, but sensitive to wind gusts

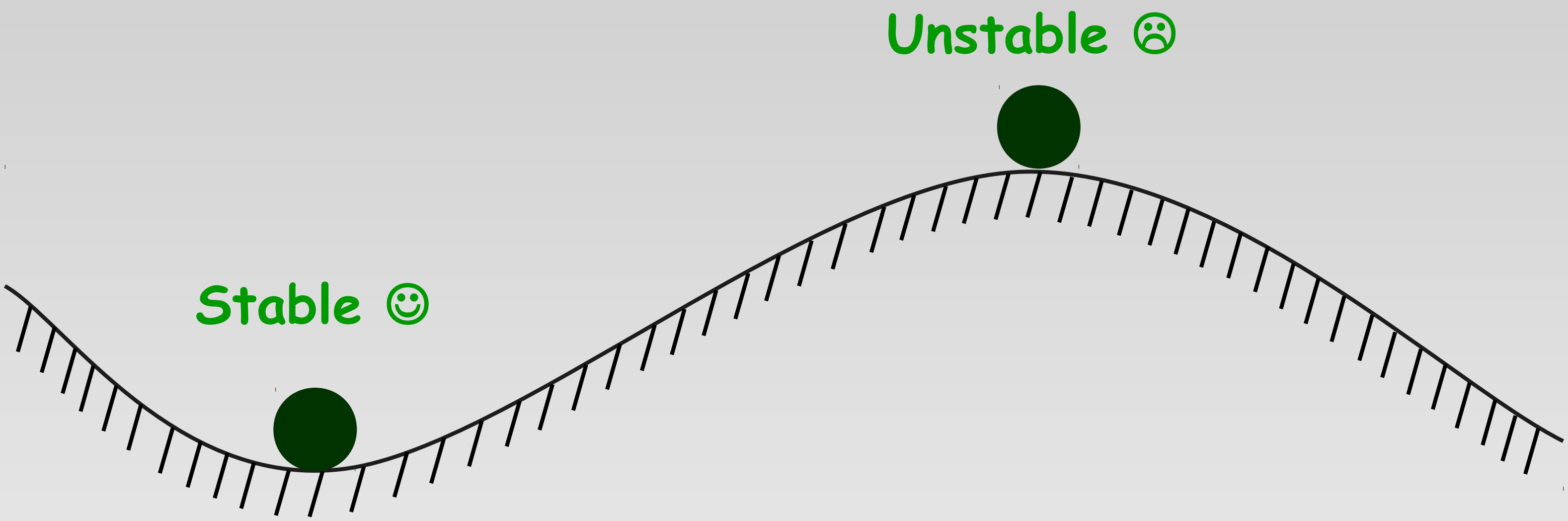
CG positioned at the NP
→ The wind vane rotates indefinitely
→ Unstable

CG behind the NP
→ The wind vane is stable… in the wrong direction

The Neutral Point is the rear limit for the CG

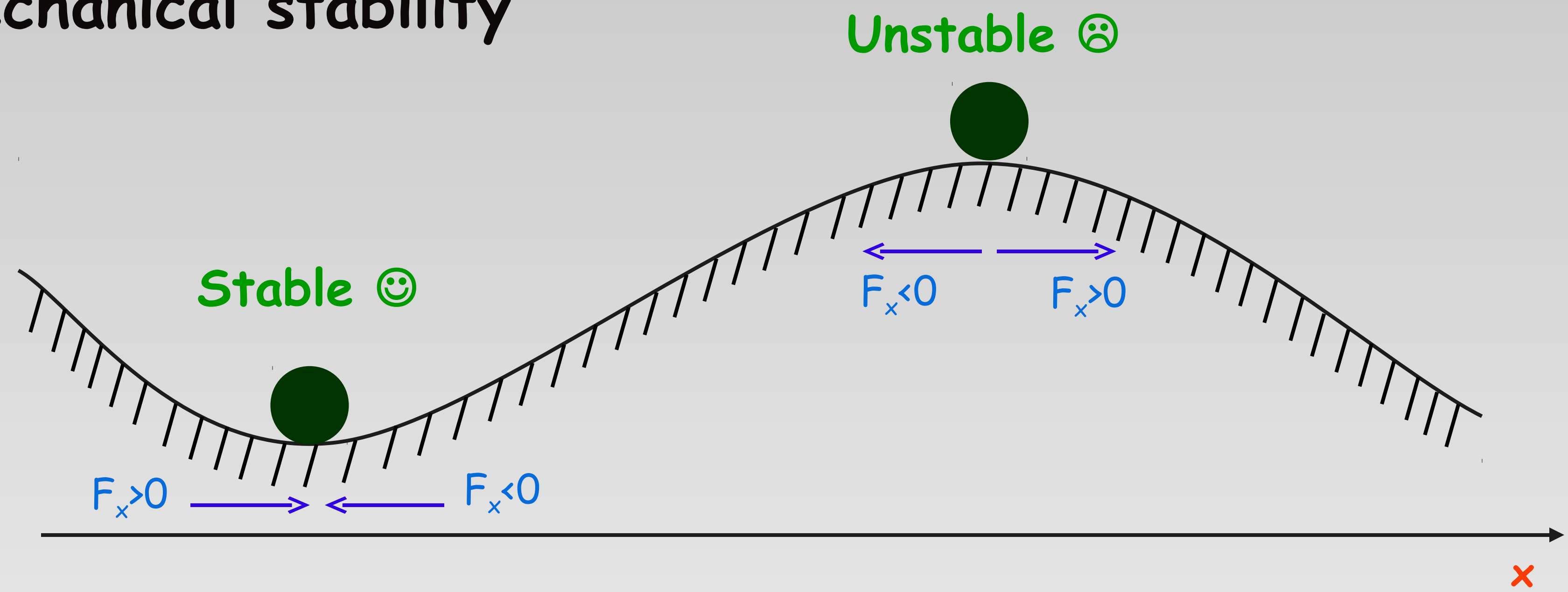
2nd principle : Forward of the NP, the CG thou shall position

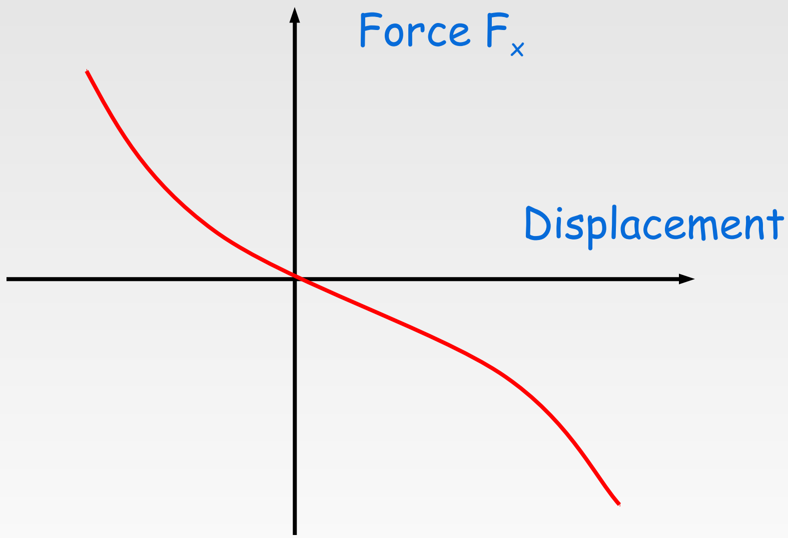
A preliminary note : Equilibrium is not stability !

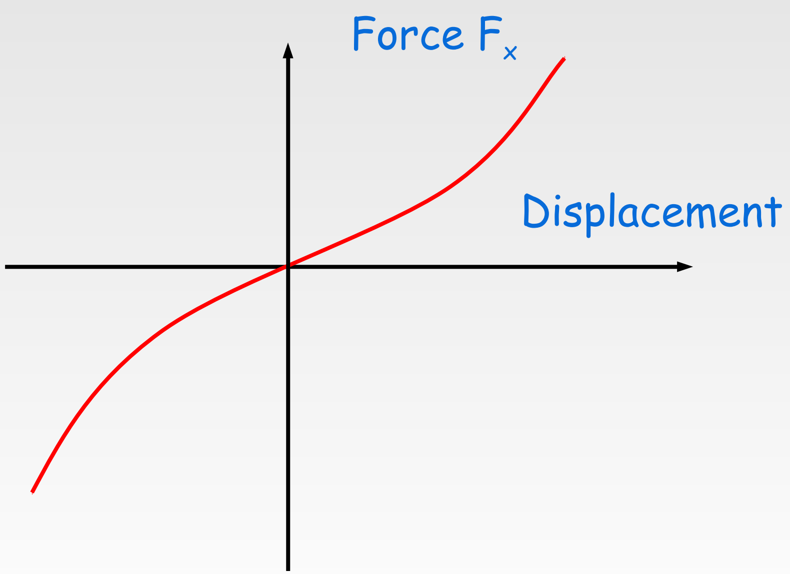


Both positions are at equilibrium, only one is stable

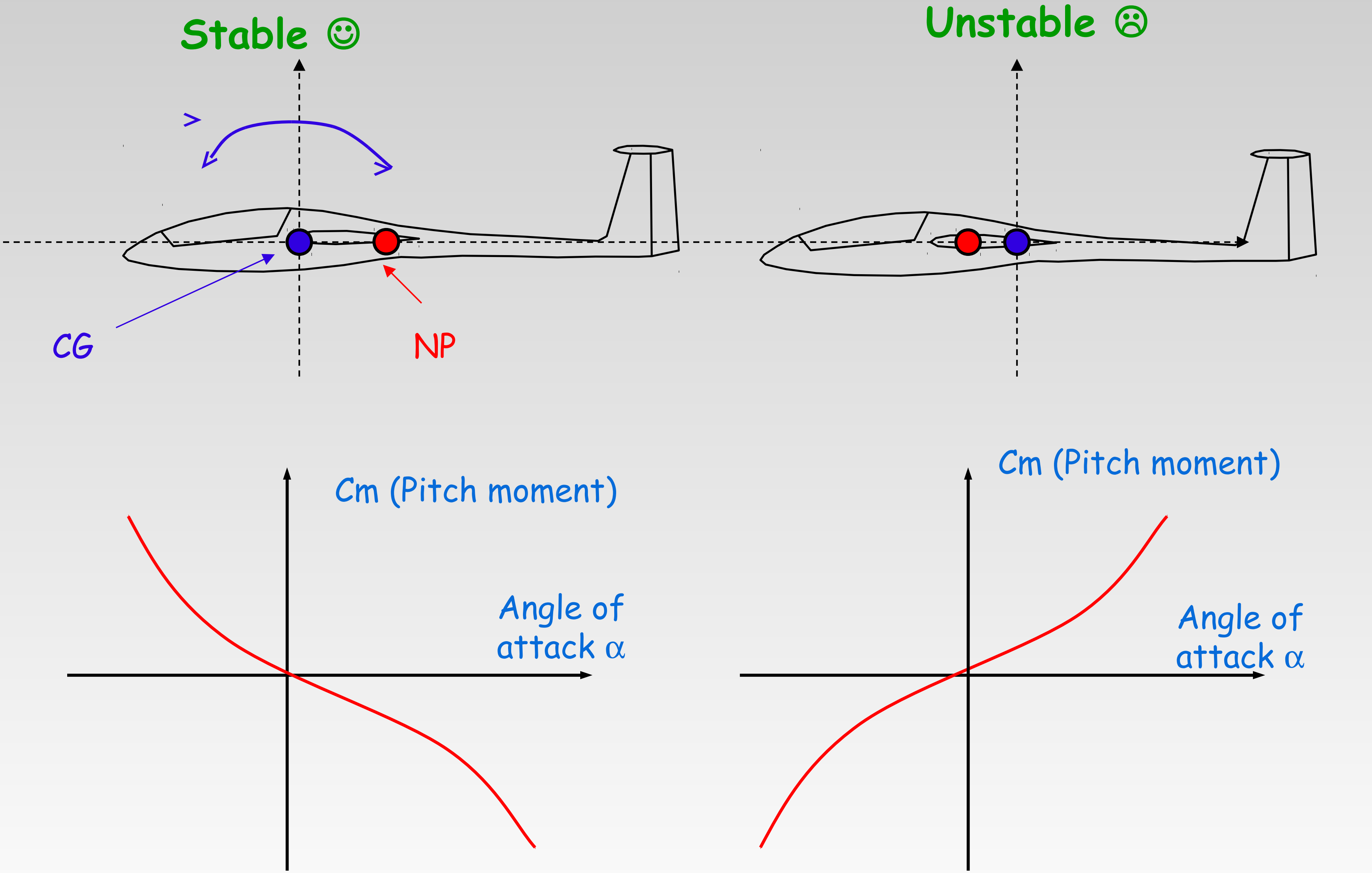
Mechanical stability





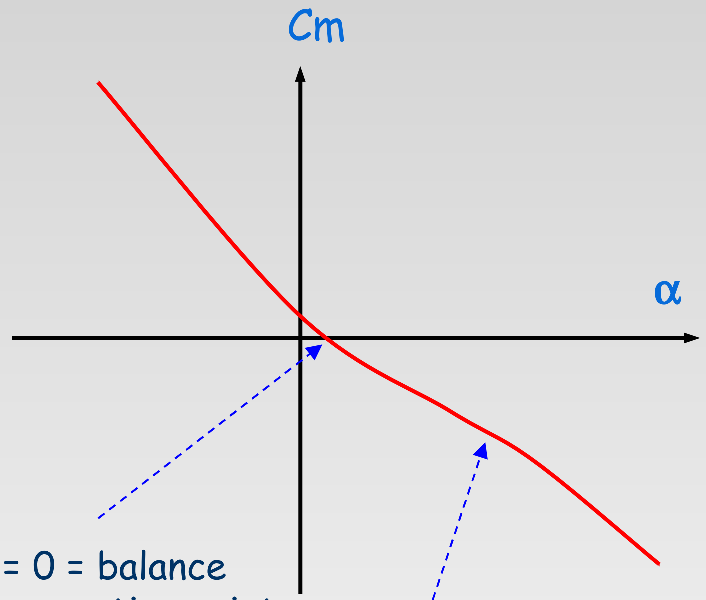


Aerodynamic stability



Understanding the polars Cm = f(α) and Cl = f(Cm)

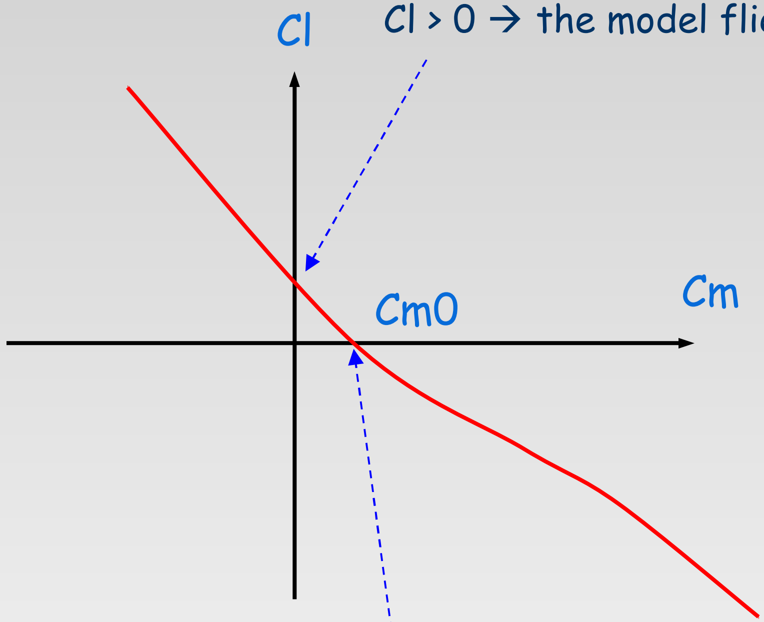
Note : Valid only for a whole plane or a flying wing



Cm = 0 = balance
= plane's operating point

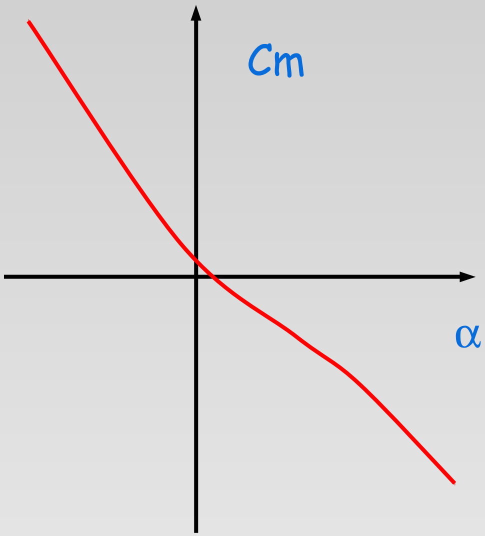
Negative slope = Stability
The curve's slope is also the strength of the stabilizing force
→ **High slope = Stable sailplane !**

Cm = 0 → balance
Cl > 0 → the model flies !

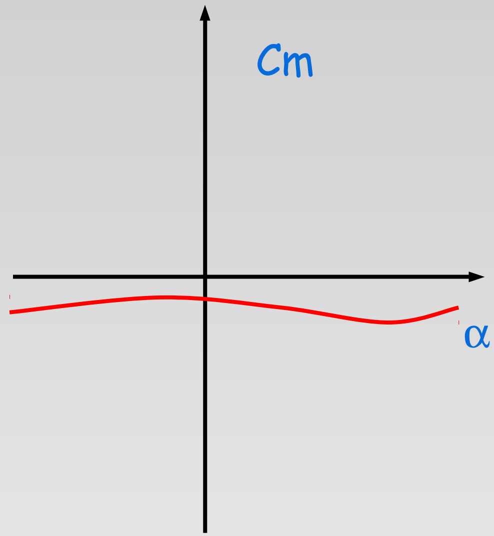


For information only :
Cm0 = Moment coefficient at zero-lift

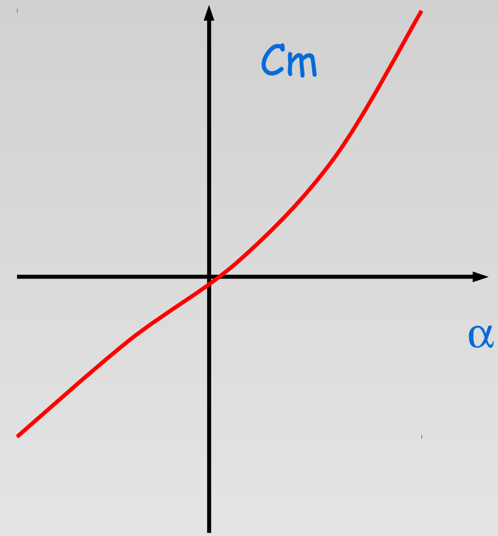
How to use XFLR5 to find the Neutral Point



Polar curve for $X_{CG} < X_{NP}$
The CG is forward of the NP
The plane is stable



Polar curve for $X_{CG} = X_{NP}$
Cm does not depend on α
The plane is unstable



Polar curve for $X_{CG} > X_{NP}$
The CG is behind the NP
The plane is stable…
The wrong way

By trial and error, find the X_{CG} value which gives the middle curve

For this value, $X_{NP} = X_{CG}$

The tail volume (1) : a condition for stability ?

First the definition

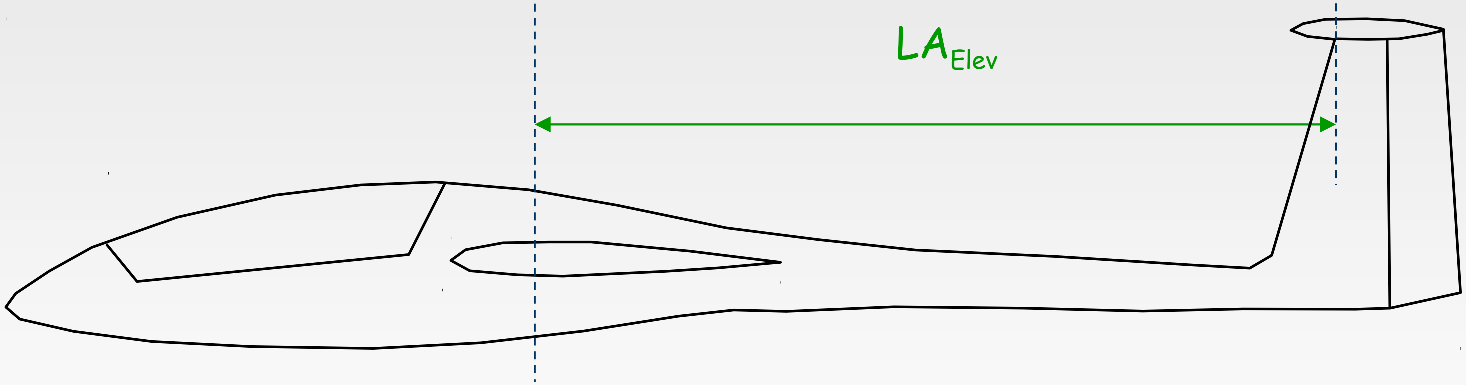
$$TV = \frac{LA_{Elev} \times Area_{Elev}}{MAC_{Wing} \times Area_{Wing}}$$

LA_{Elev} : The elevator's Lever Arm measured at the wing's and elevator's quarter chord point

MAC : The main wing's Mean Aerodynamic Chord

$Area_{Wing}$: The main wing's area

$Area_{Elev}$: The elevator's area



Tail Volume (2)

Let's write the balance of moments at the wing's quarter chord point, ignoring the elevator's self-pitching moment

$$M_{Wing} + LA_{Elev} \times Lift_{Elev} = 0$$

M_{Wing} is the wing's pitching moment around its root ¼ chord point

We develop the formula using Cl and Cm coefficients :

$$q \times Area_{Wing} \times MAC_{Wing}\, Cm_{Wing} = - LA_{Elev} \times q \times Area_{Elev} \times Cl_{Elev}$$

where q is the dynamic pressure.

Thus :

$$Cm_{Wing} = -\frac{LA_{Elev} \times Area_{Elev}}{MAC_{Wing} \times Area_{Wing}} Cl_{Elev} = -TV \times Cl_{Elev}$$

Tail Volume (3)

The elevator's influence increases with the lever arm

The elevator's influence increases with its area

$$Cm_{Wing} = -\frac{LA_{Elev} \times Area_{Elev}}{MAC_{Wing} \times Area_{Wing}} Cl_{Elev} = -TV \times Cl_{Elev}$$

The elevator has less influence as the main wing grows wider and as its surface increases

We understand now that the tail volume is a measure of the elevator's capacity to balance the wing's self pitching moment

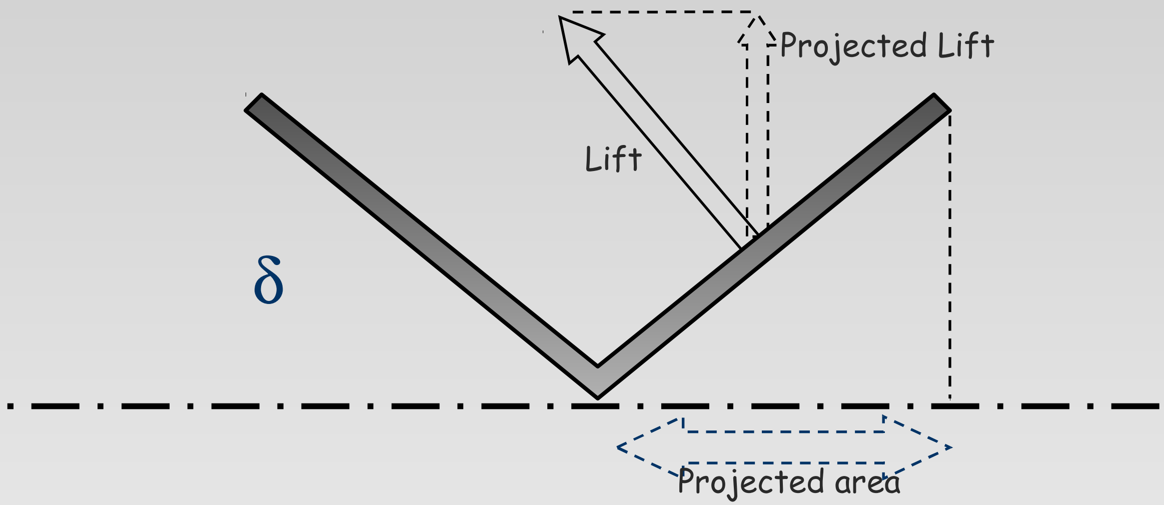
Tail Volume (4)

$$Cm_{Wing} = -\frac{LA_{Elev} \times Area_{Elev}}{MAC_{Wing} \times Area_{Wing}} Cl_{Elev} = -TV \times Cl_{Elev}$$

- The formula above tells us only that the higher the TV, the greater the elevator's influence shall be
- It does not give us any clue about the plane's stability
- It tells us nothing on the values and on the signs of Cm and Cl
- This is a necessary condition, but not sufficient : we need to know more on pitching and lifting coefficients
- Thus, an adequate value for the tail volume is not a condition sufficient for stability

A little more complicated : V-tails

The method is borrowed from Master Drela (may the aerodynamic Forces be with him)



The angle δ has a double influence:

1. It reduces the surface projected on the horizontal plane
2. It reduces the projection of the lift force on the vertical plane

… after a little math:

$$\text{Effective\_area} = \text{Area}_{Elev} \times \cos^2\delta$$

$$TV = \frac{LA_{Elev} \times Area_{Elev} \times \cos^2\delta}{MAC_{Wing} \times Area_{Wing}}$$

The Static Margin : a useful concept

- First the definition

$$SM = \frac{X_{NP} - X_{CG}}{MAC_{Wing}}$$

- A positive static margin is synonym of stability
- The greater is the static margin, the more stable the sailplane will be
- We won't say here what levels of static margin are acceptable… too risky… plenty of publications on the matter also
- Each user should have his own design practices
- Knowing the NP position and the targeted SM, the CG position can be deduced…= X_{NP} - MAC x SM
- …without guarantee that this will correspond to a positive lift nor to optimized performances

How to use XFLR5 to position the CG

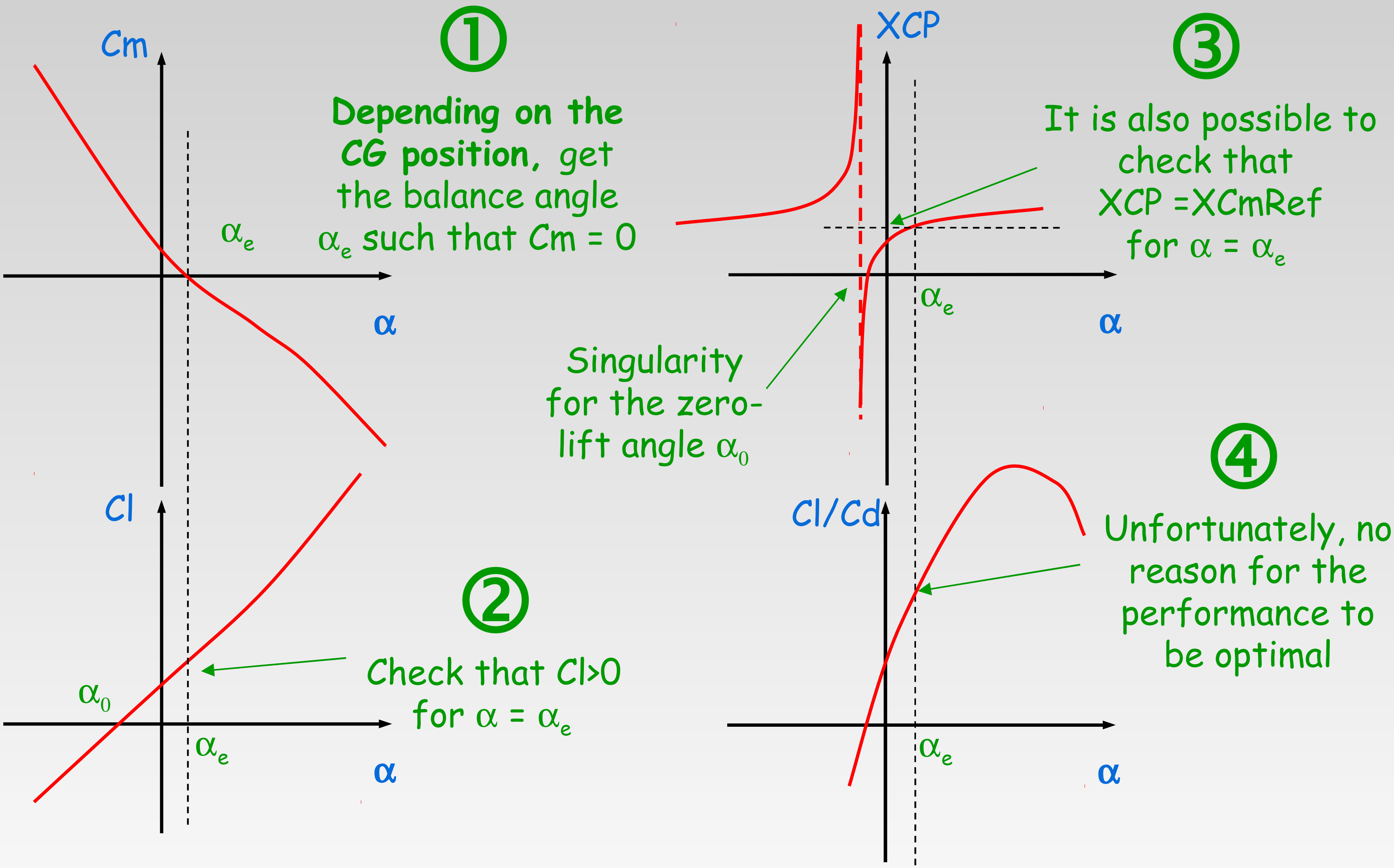
- **Idea N°1 : the most efficient**
  - Forget about XFLR5
  - Position the CG at 30-35% of the Mean Aero Chord
  - Try soft hand launches in an area with high grass
  - Move progressively the CG backwards until the plane glides normally
  - For a flying wing
    - Start at 15%
    - Set the ailerons up 5°-10°
    - Reduce progressively aileron angle and move the CG backwards
  - Finish off with the dive test

→ Works every time !

How to use XFLR5 to position the CG

- **Idée N°2 : Trust the program**
  - Re-read carefully the disclaimer
  - Find the Neutral Point as explained earlier
  - Move the CG forward from the NP…
  - … to achieve a slope of Cm = f(α) comparable to that of a model which flies to your satisfaction, or
  - … to achieve an acceptable static margin
  - Go back to Idea N°1 and perform a few hand launches

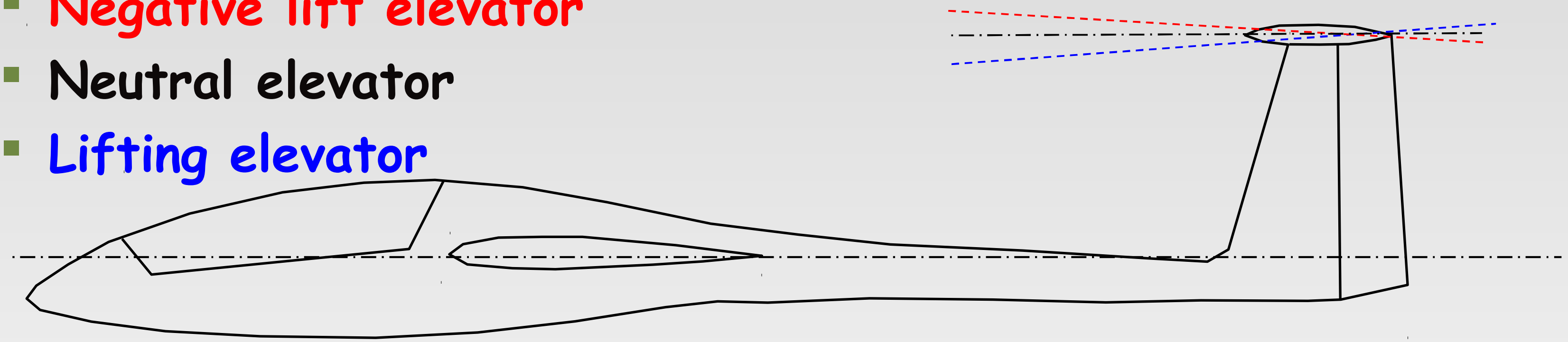
Summarizing on the 4-graph view of XFLR5



Iterations are required to find the best compromise

Consequences of the incidence angle

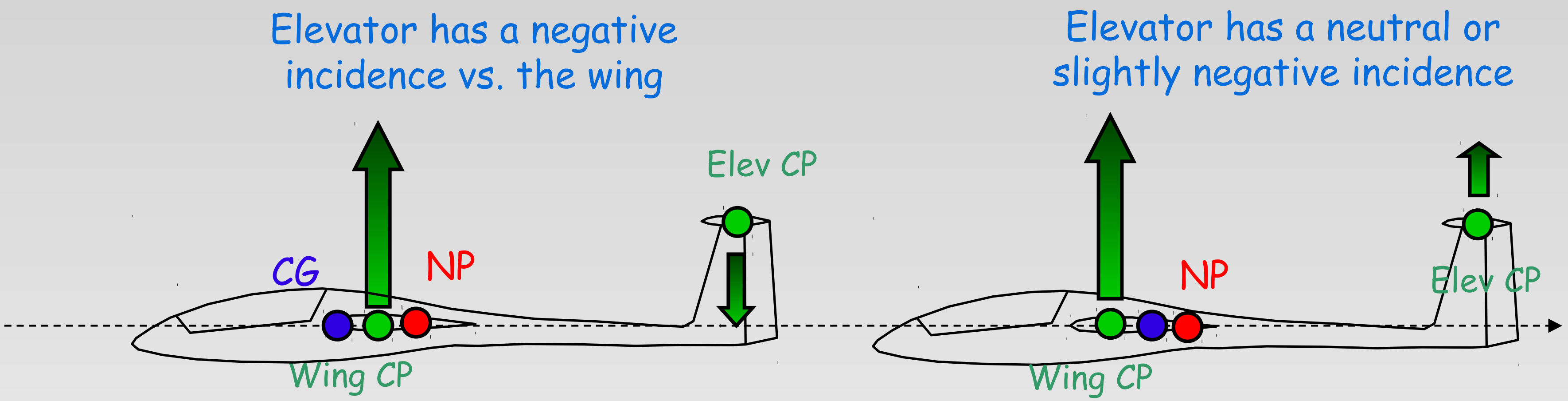
- To achieve lift, the wing must have an angle of attack greater than its zero-lift angle
- This angle of attack is achieved by the balance of wing and elevator lift moments about the CG
- Three cases are possible
  - Negative lift elevator
  - Neutral elevator
  - Lifting elevator



- Each case leads to a different balanced angle of attack
- For French speakers, read Matthieu's great article on http://pierre.rondel.free.fr/Centrage_equilibrage_stabilite.pdf

Elevator Incidence and CG position

- The elevator may have a positive or negative lift



- Both configurations are possible
- The CG will be forward of the wing's CP for an elevator with negative lift
- "Within the acceptable range of CG position, the glide ratio does not change much" (M. Scherrer 2006)

The case of Flying Wings

- No elevator
- The main wing must achieve its own stability
- Two options
  - Self stable foils
  - Negative washout at the wing tip

Self-Stable Foils

- The notion is confusing : The concept covers those foils which make a wing self-stable, without the help of a stabilizer
- Theory and analysis tell us that a foil's Neutral Point is at distance from the leading edge = 25% x chord
- But then… all foils are self-stable ??? All that is required is to position the CG forward of the NP
- What's the difference between a so-called self-stable foil and all of the others ???

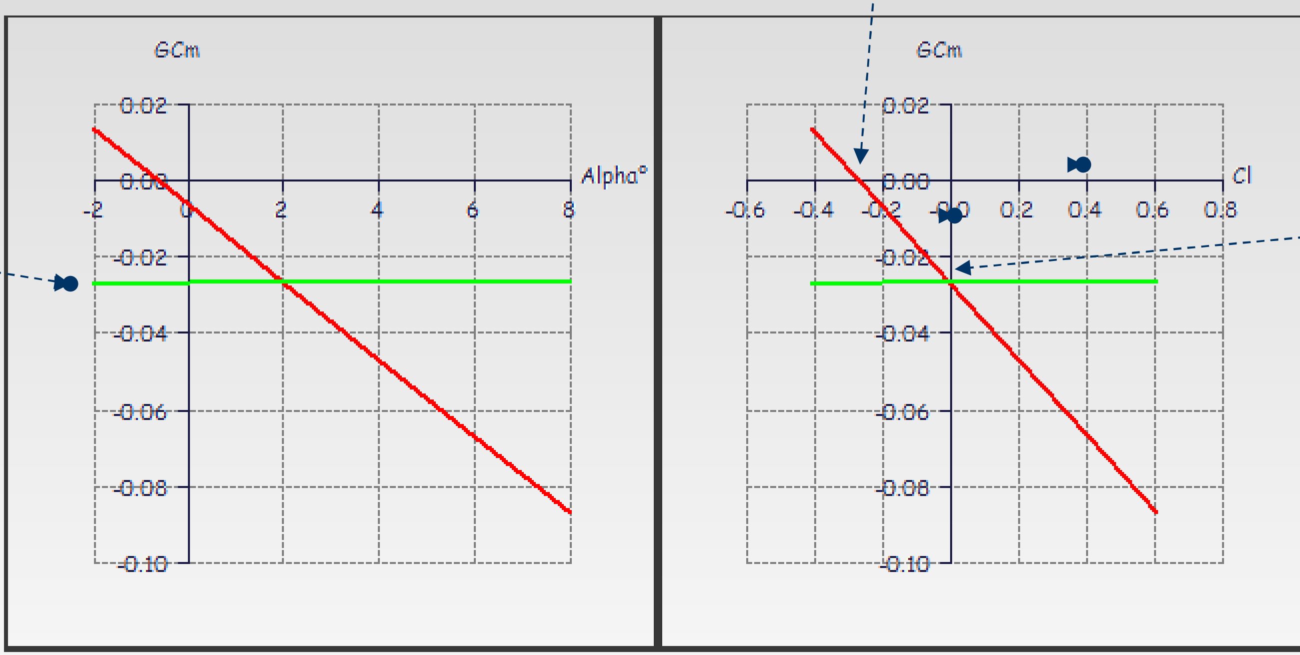
→ Let's explore it with the help of XFLR5

A classic foil

Consider a rectangular wing with uniform chord =100 mm, with a NACA 1410 foil reputedly not self-stable

Unfortunately, at zero pitching moment, the lift is negative, the wing does not fly. That's the problem…

Calculations confirm that the NP is at 25% of the chord



Straight wing NACA 1410 no twist
T1-10.0 m/s-VLM1- 15.00mm
T1-10.0 m/s-VLM1- 25.00mm

It is usually said of these airfoils that their zero-lift moment coefficient is negative
Cm0 < 0

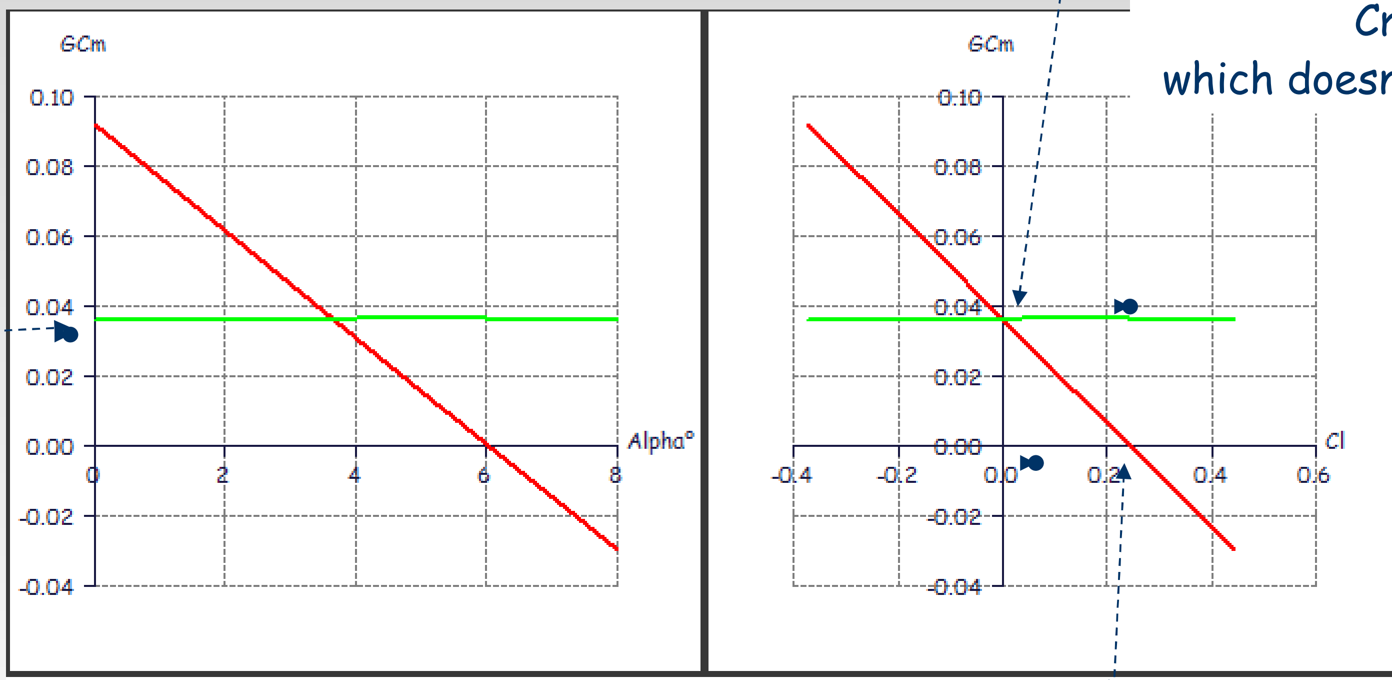
Note : this analysis can also be done in non-linear conditions with XFoil

A self-stable foil

Consider the same rectangular wing with chord 100mm, with an Eppler 186 foil known to be self-stable

The NP is still at 25% of the chord

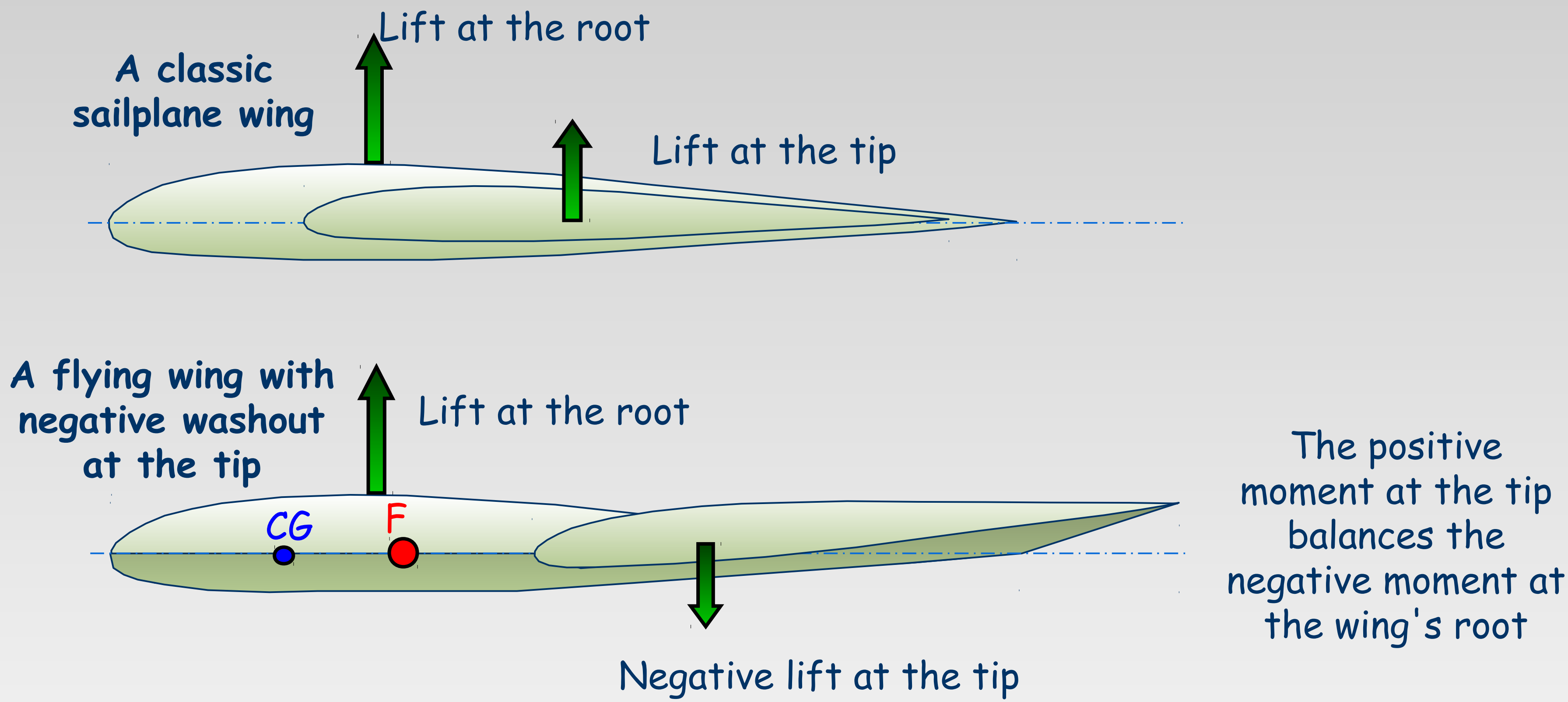
It is usually said of these airfoils that "the zero-lift moment is positive", $Cm_0 > 0$ which doesn't tell us much



Straight wing Eppler 186 no twist
T1-10.0 m/s-VLM1- 10.00mm
T1-10.0 m/s-VLM1- 25.00mm

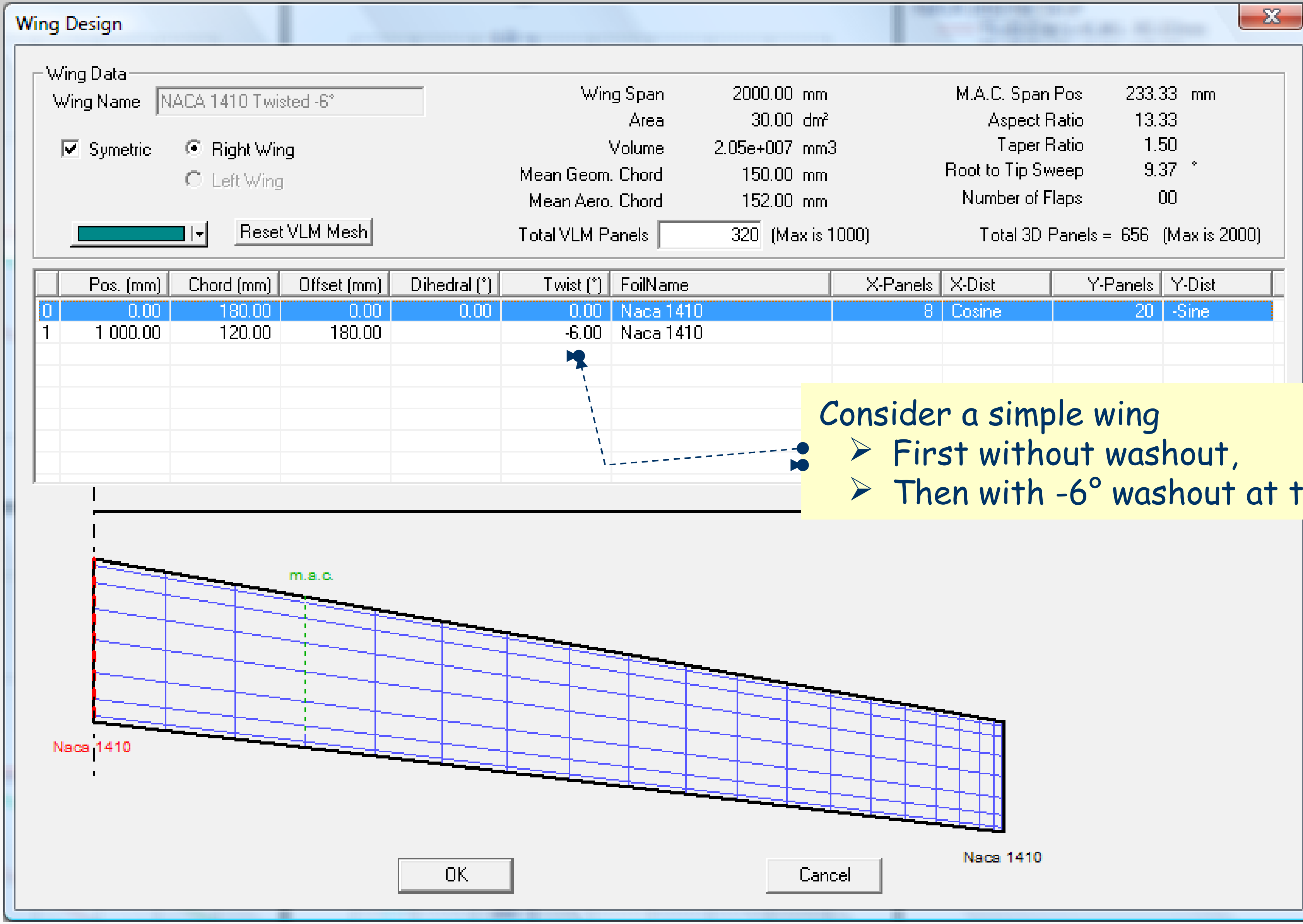
It would be more intuitive to say "the zero-moment lift is positive" : $Cl_0 > 0$, the wing flies!

A more modern way to create a self-stable wing



- The consequence of the negative lift at the tip is that the total lift will be less than with the classic wing
- Let's check all this with XFLR5

Model data

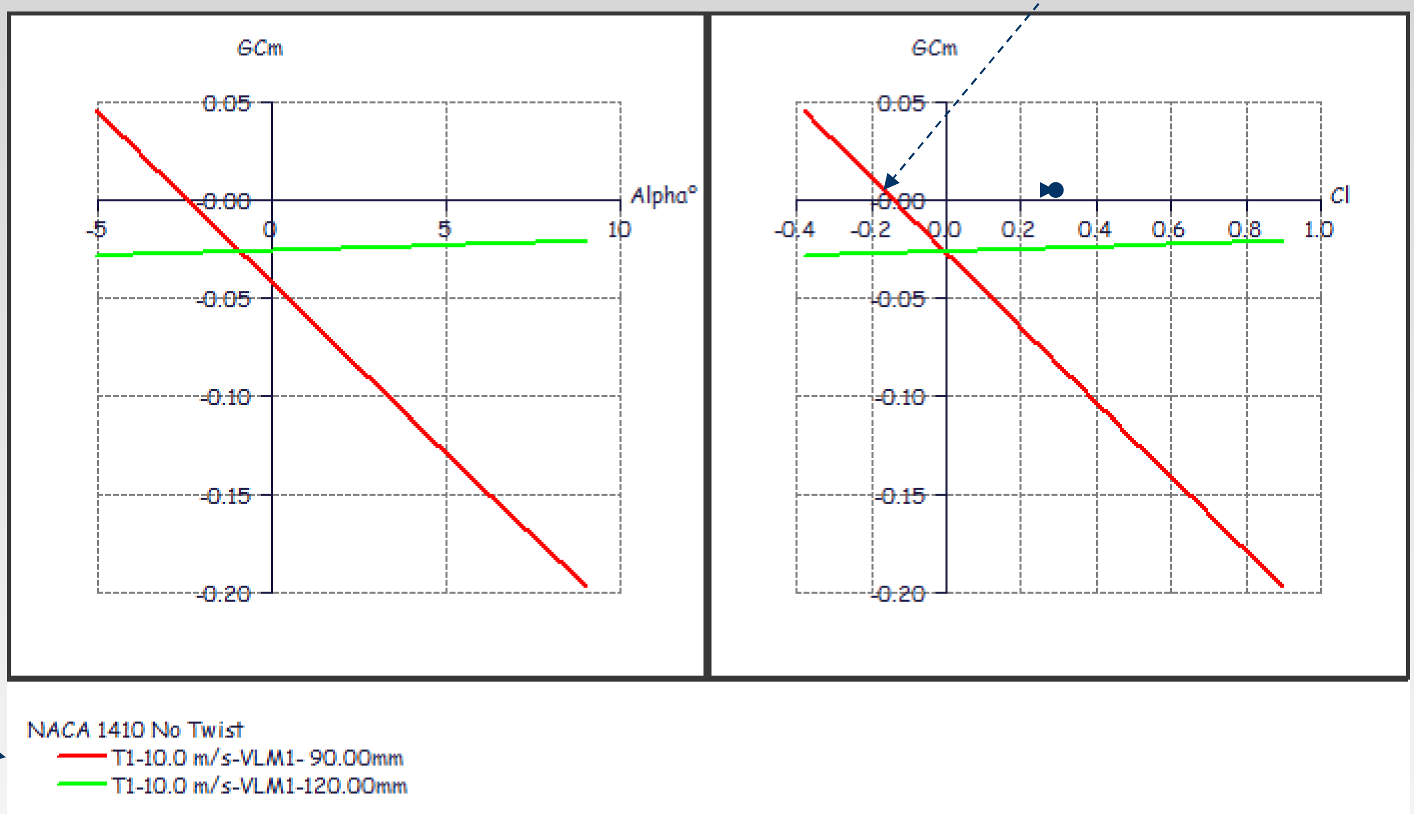


Consider a simple wing

- First without washout,
- Then with -6° washout at tip

Wing without washout

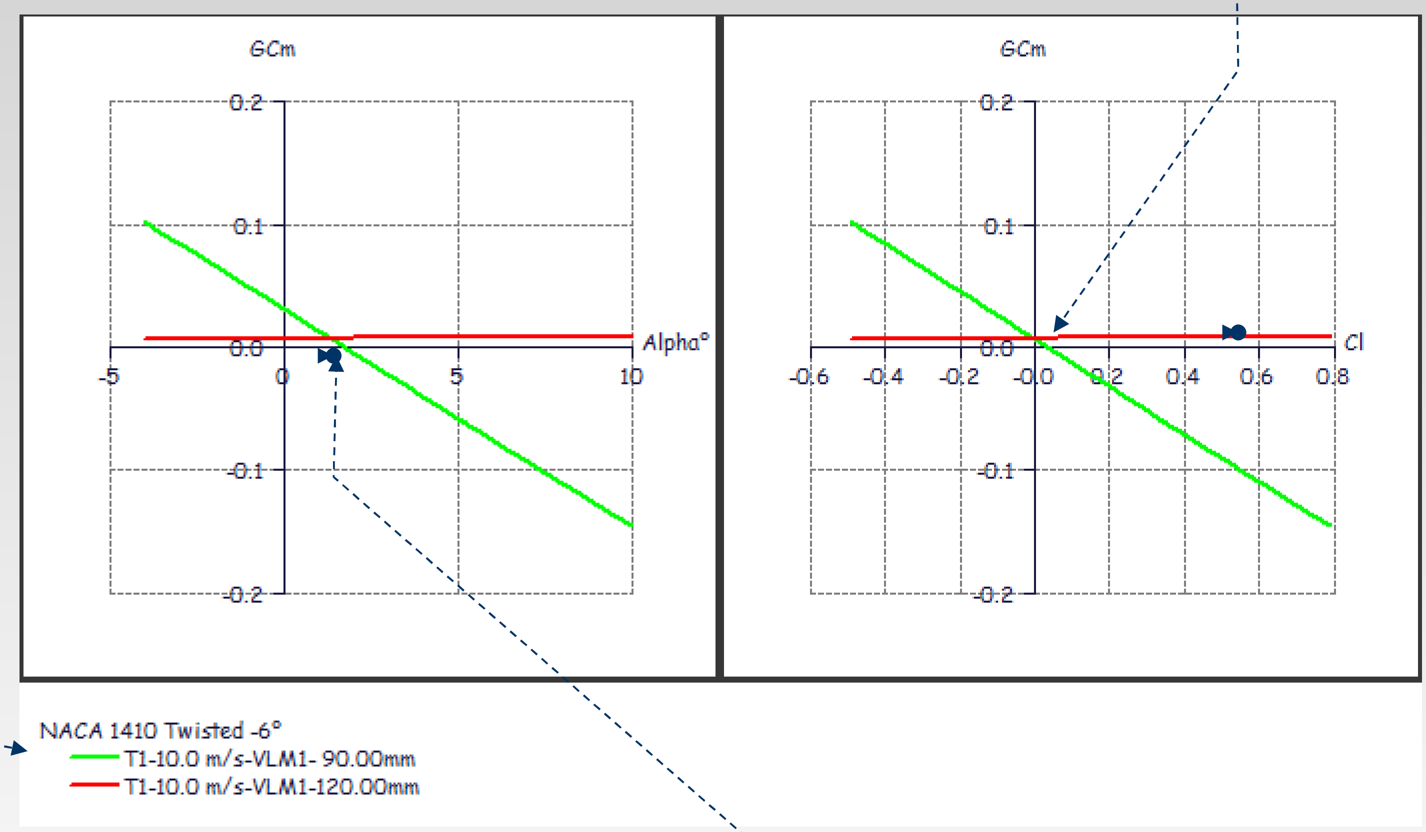
Unfortunately, at zero pitching moment, the lift is negative (Cl<0) : the wing does not fly



Consider a static margin = 10%

Wing with washout

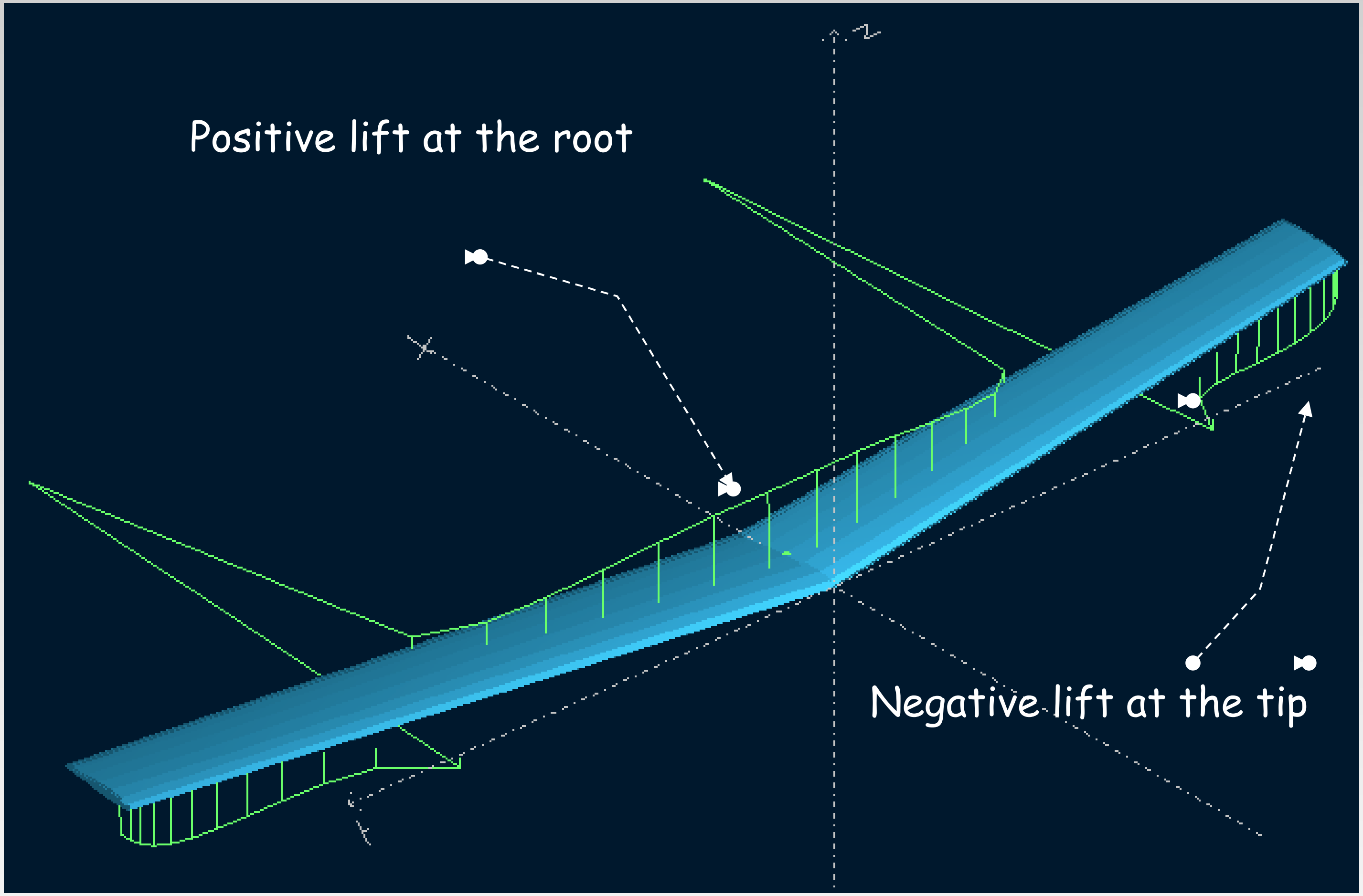
At zero pitching moment, the lift is slightly positive :
It flies !



Consider a static margin = 10%

Let's visualize in the next slide the shape of the lift for the balanced a.o.a $\alpha_e=1.7°$

Lift at the balanced a.o.a



Part of the wing lifts the wrong way : a flying wing exhibits low lift

Stability and Control analysis

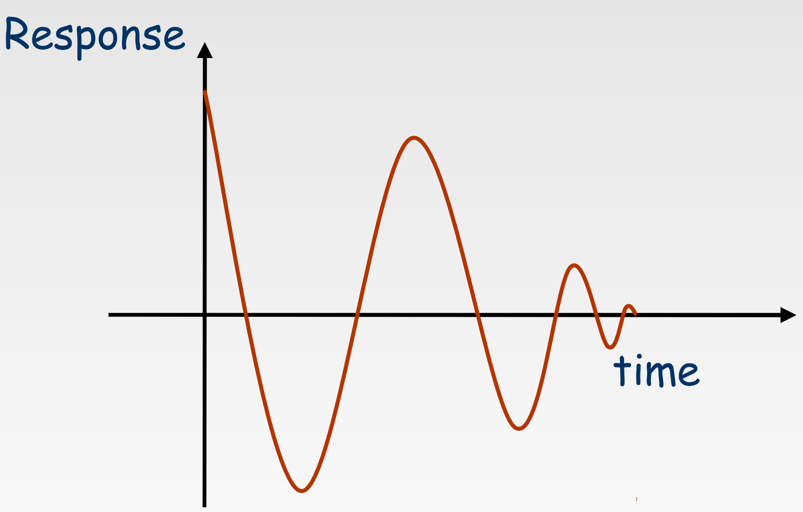
So much for performance… but what about stability and control ?

What it's all about

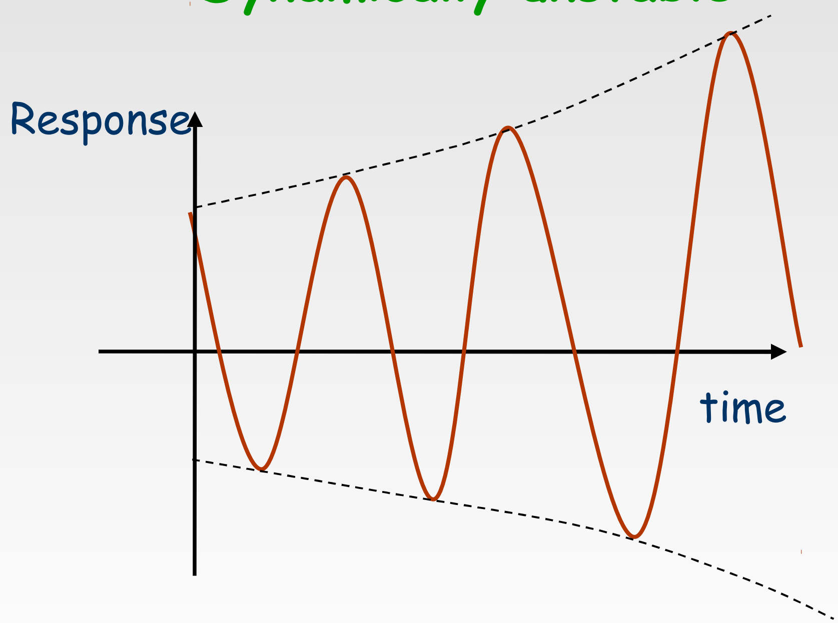
- Our model aircraft needs to be adjusted for performance, but needs also to be stable and controllable.
  - Stability analysis is a characteristic of "hands-off controls" flight
  - Control analysis measures the plane's reactions to the pilot's instructions
- To some extent, this can be addressed by simulation
- An option has been added in XFLR5 v6 for this purpose

Static and Dynamic stability

Dynamically stable



Dynamically unstable



Sailplane stability

- A steady "static" state for a plane would be defined as a constant speed, angle of attack, bank angle, heading angle, altitude, etc.
- Difficult to imagine
- Inevitably, a gust of wind, an input from the pilot will disturb the plane
- The purpose of Stability and Control Analysis is to evaluate the dynamic stability and time response of the plane for such a perturbation
- In the following slides, we refer only to dynamic stability

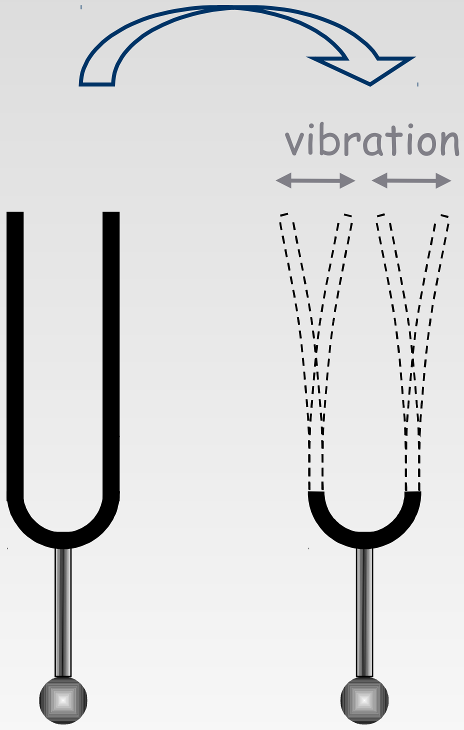
Natural modes

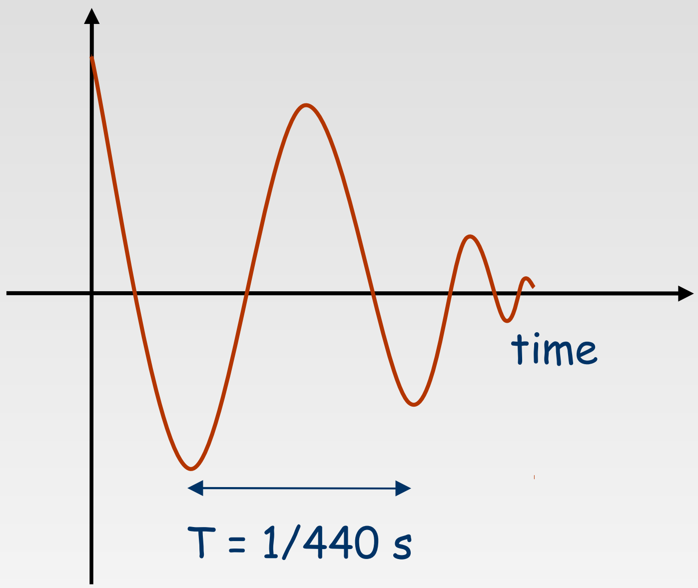
- Physically speaking, when submitted to a perturbation, a plane tends to respond on "preferred" flight modes
- From the mathematic point of view, these modes are called "Natural modes" and are described by
  - an eigenvector, which describes the modal shape
  - an eigenvalue, which describes the mode's frequency and its damping

Natural modes - Mechanical

- Example of the tuning fork

Shock perturbation
→ preferred response on A note
= 440 Hz

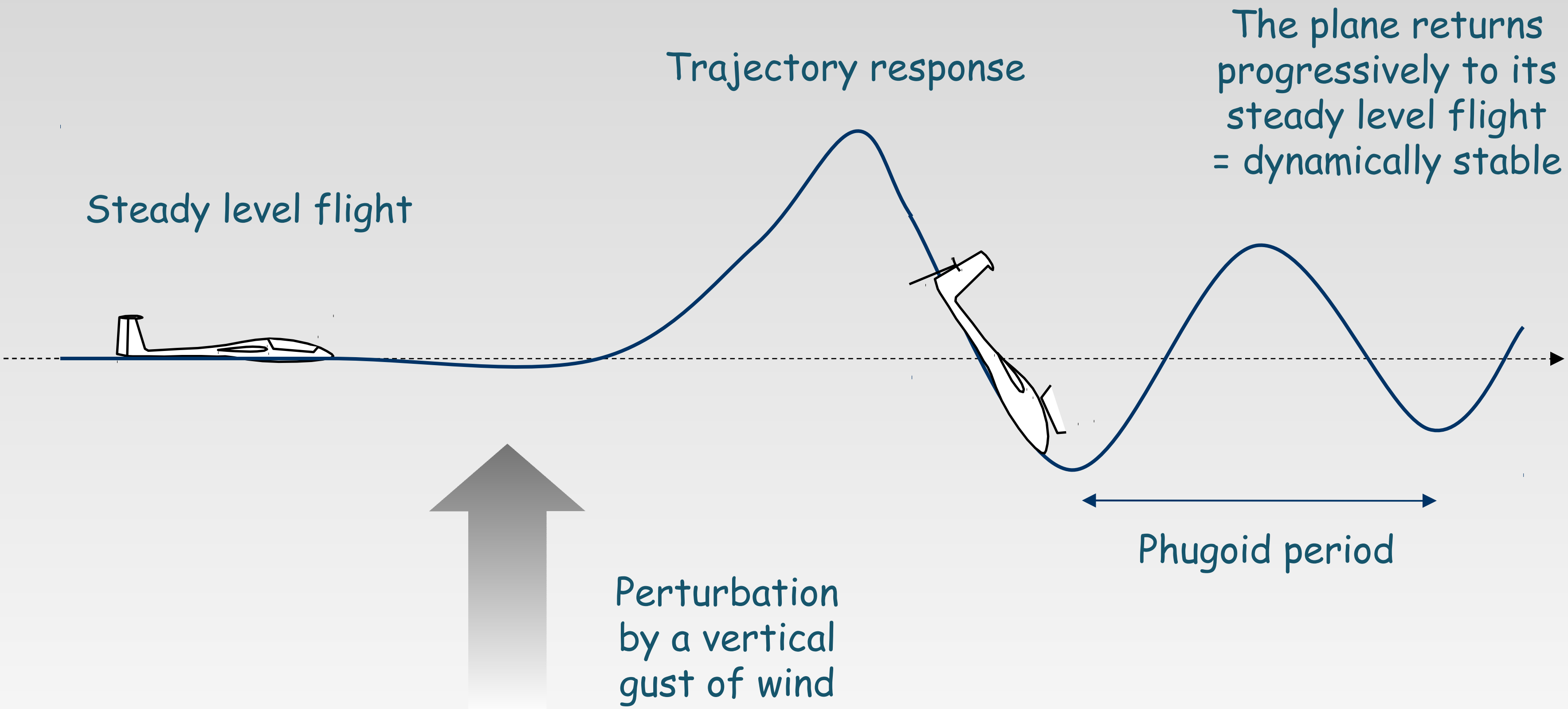




The sound decays with time
The fork is dynamically stable… not really a surprise

Natural modes - Aerodynamic

- Example of the phugoid mode



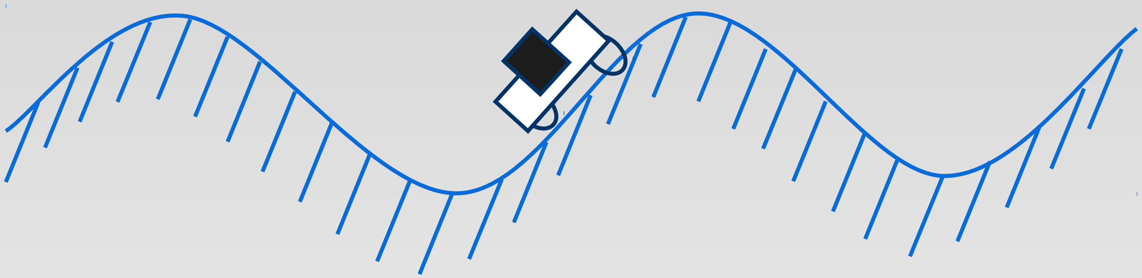
The 8 aerodynamic modes

- **A well designed plane will have 4 natural longitudinal modes and 4 natural lateral modes**

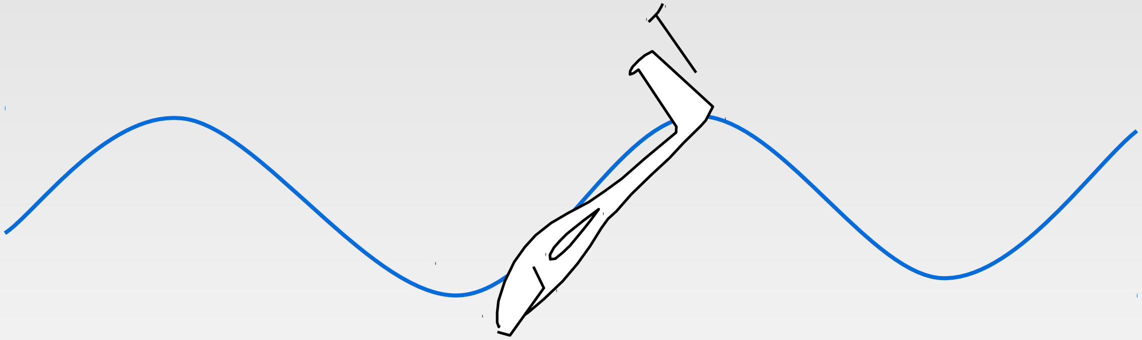
Longitudinal	Lateral
2 symmetric phugoid modes	1 spiral mode
2 symmetric short period modes	1 roll damping mode
	2 Dutch roll modes

The phugoid

... is a macroscopic mode of exchange between the Kinetic and Potential energies



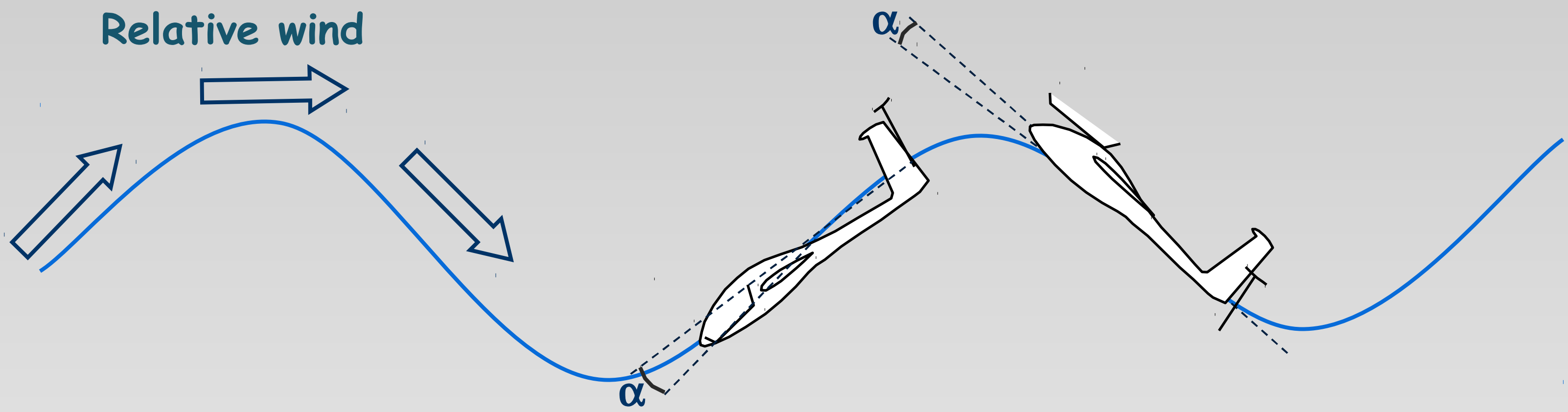
Russian Mountains : Exchange is made by the contact force

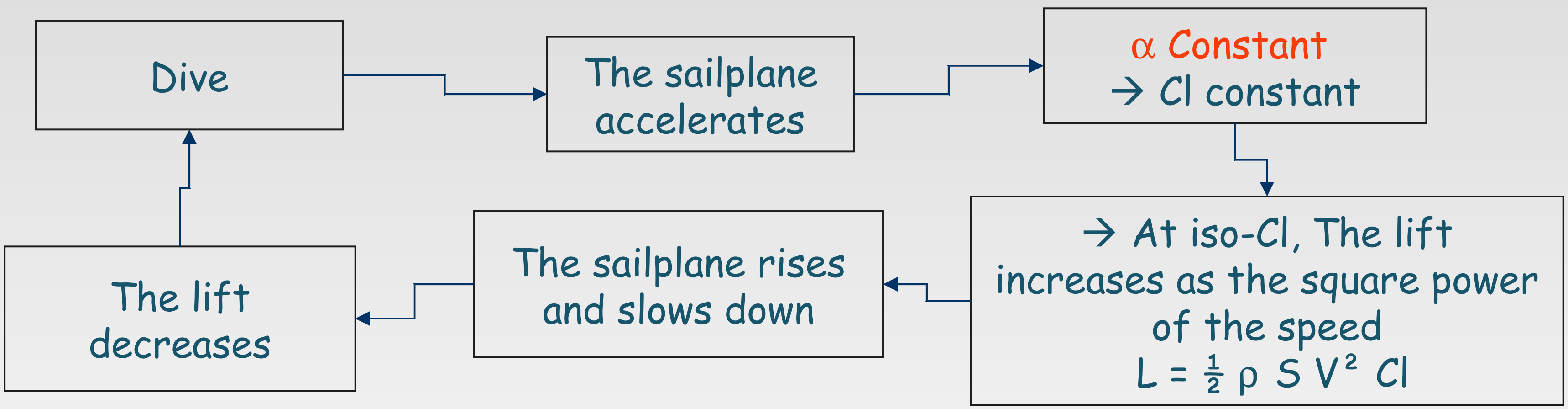


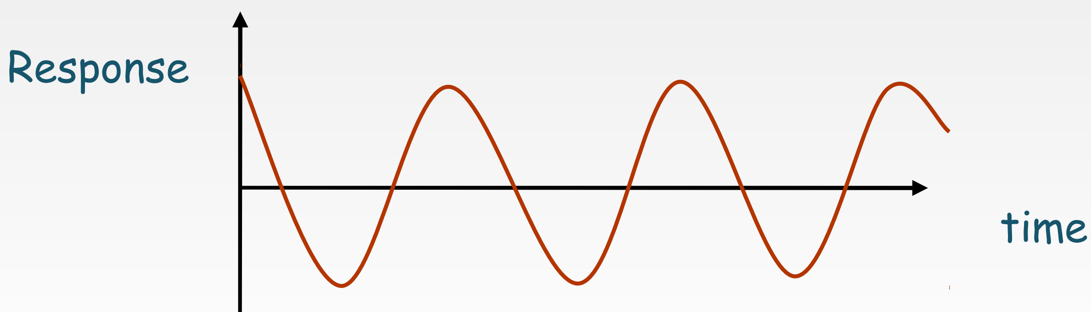
Aerodynamic : Exchange is made by the lift force

Slow, lightly damped, stable or unstable

The mechanism of the phugoid

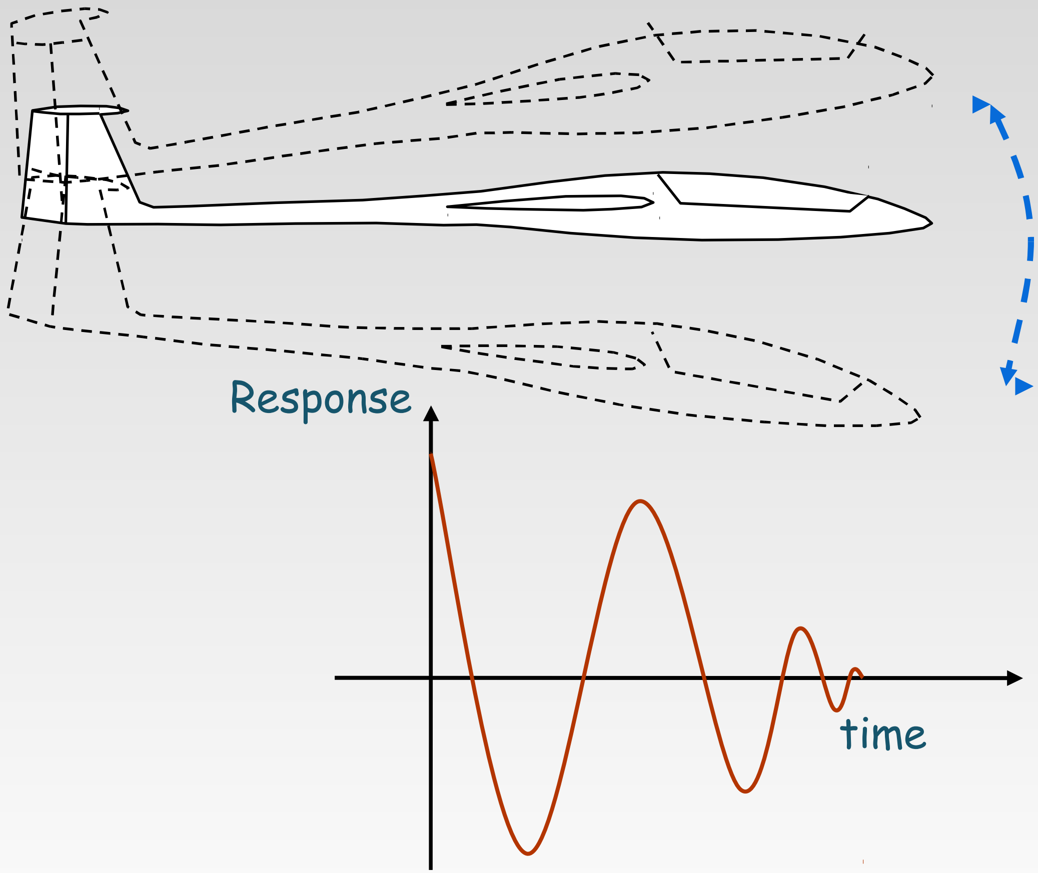






The short period mode

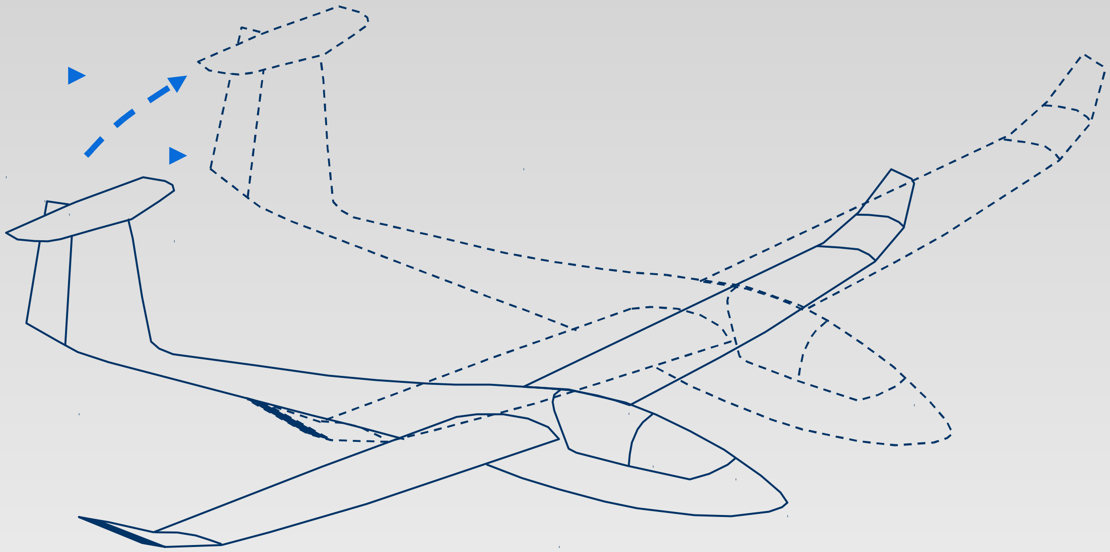
- Primarily vertical movement and pitch rate in the same phase, usually high frequency, well damped



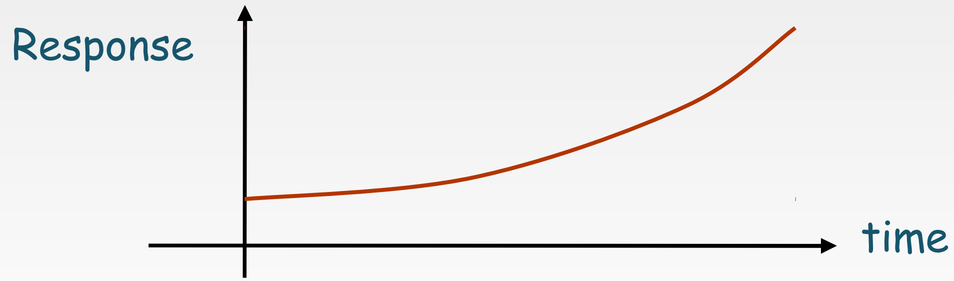
The mode's properties are primarily driven by the stiffness of the negative slope of the curve $Cm=f(\alpha)$

Spiral mode

- **Primarily heading, non-oscillatory, slow, generally unstable**



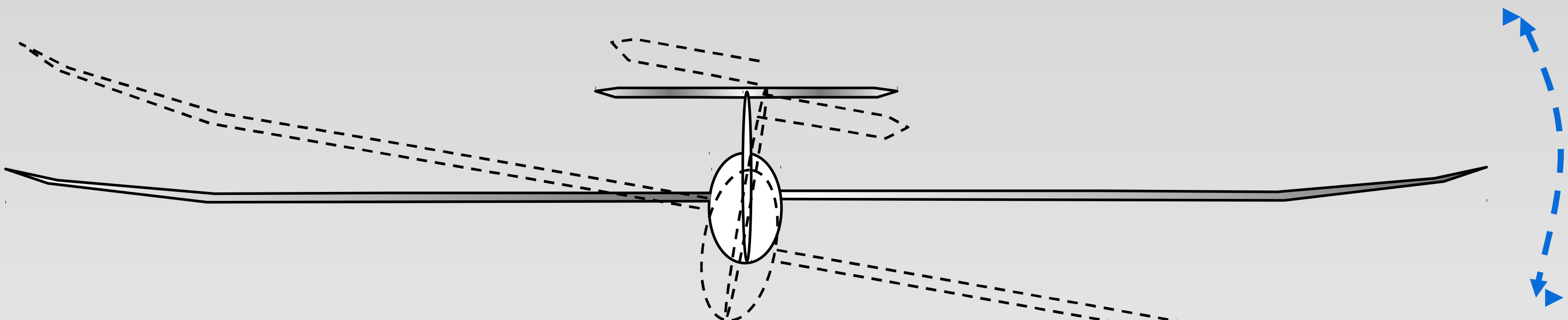
The mode is initiated by a rolling or heading disturbance. This creates a positive a.o.a. on the fin, which tends to increase the yawing moment



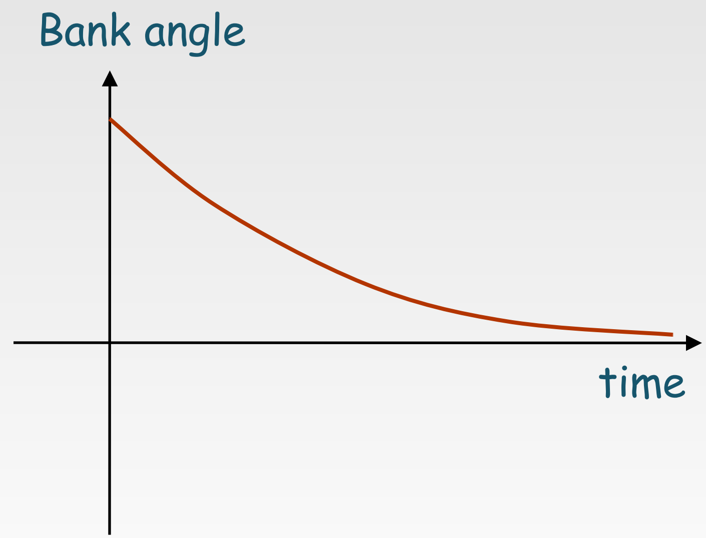
Requires pilot input to prevent divergence !

Roll damping

- **Primarily roll, stable**

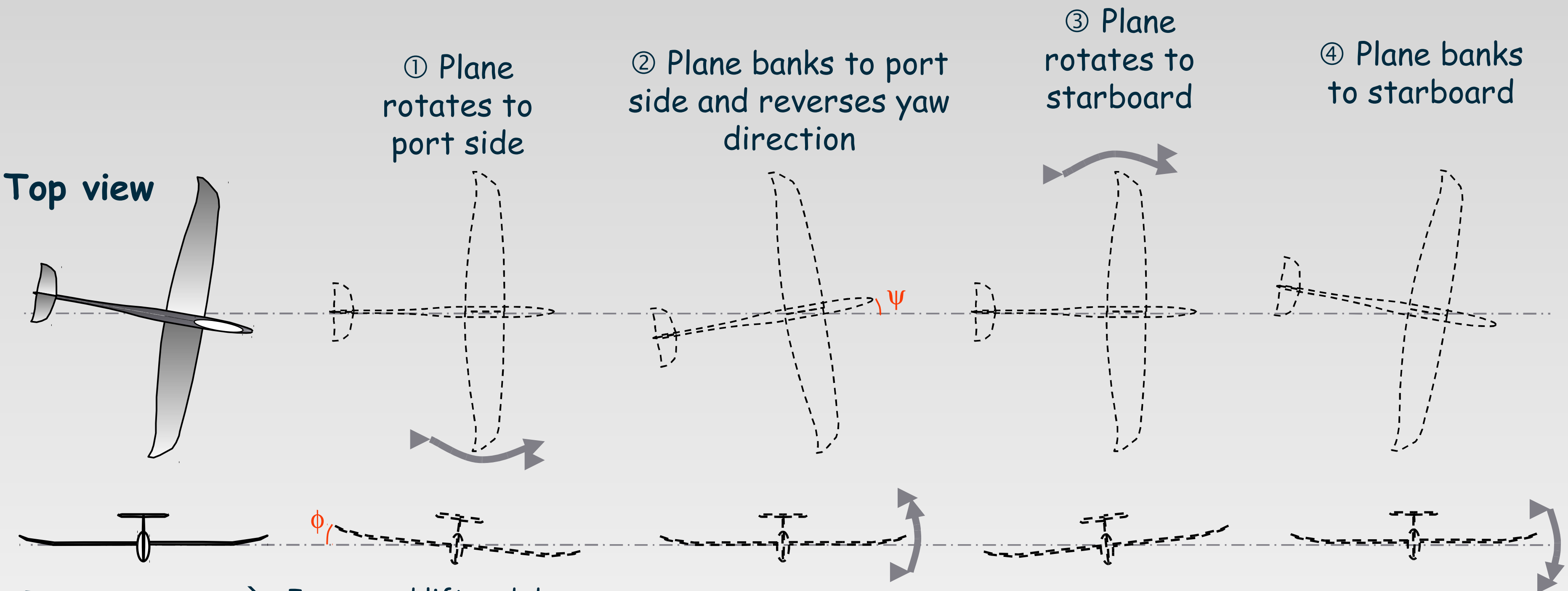


1. Due to the rotation about the x-axis, the wing coming down sees an increased a.o.a., thus increasing the lift on that side. The symmetric effect decreases the lift on the other side.
2. This creates a restoring moment opposite to the rotation, which tends to damp the mode



Dutch roll

- The Dutch roll mode is a combination of yaw and roll, phased at 90°, usually lightly damped

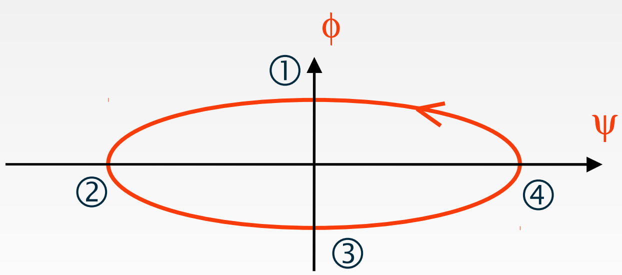


Rear view

→ Increased lift and drag on starboard side, decreased lift and drag on port side

→ The lift difference creates a bank moment to port side

→ The drag difference creates a yawing moment to starboard



Modal response for a reduced scale plane

- During flight, a perturbation such as a control input or a gust of wind will excite all modes in different proportions :
  - Usually, the response on the short period and the roll damping modes, which are well damped, disappear quickly
  - The response on the phugoid and Dutch roll modes are visible to the eye
  - The response on the spiral mode is slow, and low in magnitude compared to other flight factors.
    It isn't visible to the eye, and is corrected unconsciously by the pilot

Modal behaviour

- **Some modes are oscillatory in nature…**
  - Phugoid,
  - Short period
  - Dutch roll

Defined by

1. a "mode shape" or eigenvector
2. a natural frequency
3. a damping factor

- **…and some are not**
  - Roll damping
  - Spiral

Defined by

1. a "mode shape" or eigenvector
2. a damping factor

The eigenvector

- In mathematical terms, the eigenvector provides information on the amplitude and phase of the flight variables which describe the mode,
- In XFLR5, the eigenvector is essentially analysed visually, in the 3D view

- A reasonable assumption is that the longitudinal and lateral dynamics are independent and are described each by four variables

The four longitudinal variables

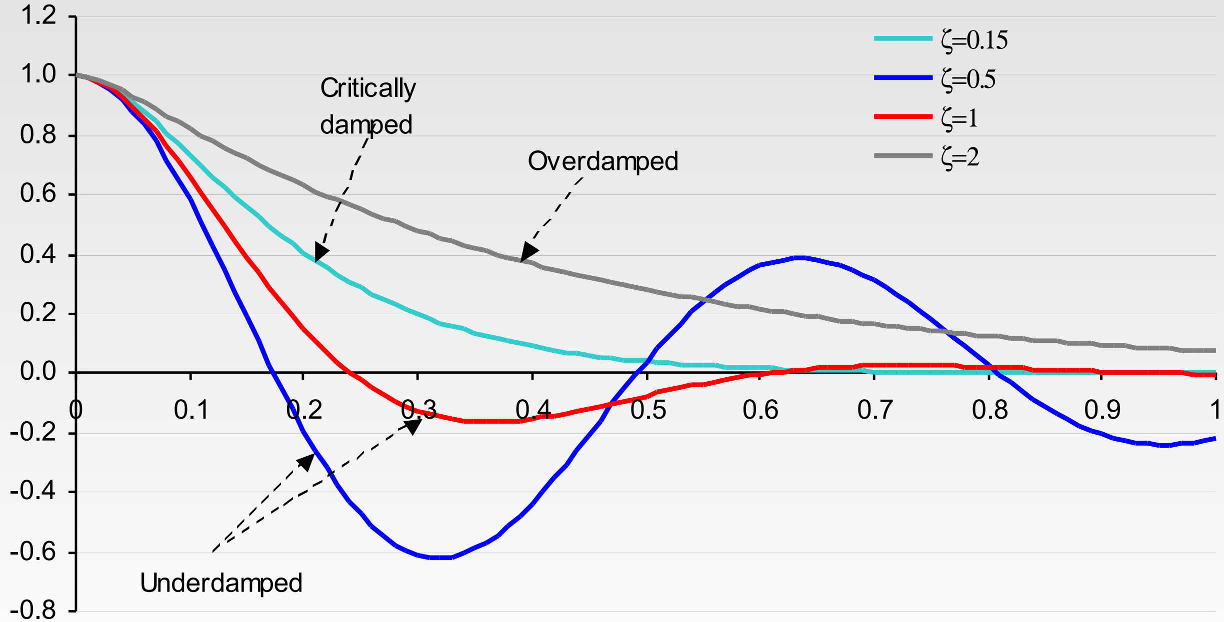
- The longitudinal behaviour is described by
  - The axial and vertical speed variation about the steady state value $V_{inf} = (U_0,0,0)$
    - $u = dx/dt - U_0$
    - $w = dz/dt$
  - The pitch rate $q = d\theta/dt$
  - The pitch angle θ
- Some scaling is required to compare the relative size of velocity increments "u" and "w" to a pitch rate "q" and to an angle "θ "
- The usual convention is to calculate
  - $u' = u/U_0$, $w' = w/U_0$, $q' = q/(2U_0/mac)$,
  - and to divide all components such that $\theta = 1$

The four lateral variables

- The longitudinal behaviour is described by four variables
  - The lateral speed variation v = dy/dt about the steady state value $V_{inf} = (U_0,0,0)$
  - The roll rate $p = d\phi/dt$
  - The yaw rate $r = d\psi/dt$
  - The heading angle ψ
- For lateral modes, the normalization convention is
  - $v' = u/U_0$, $p' = p/(2U_0/\text{span})$, $r' = r/(2U_0/\text{span})$,
  - and to divide all components such that $\psi = 1$

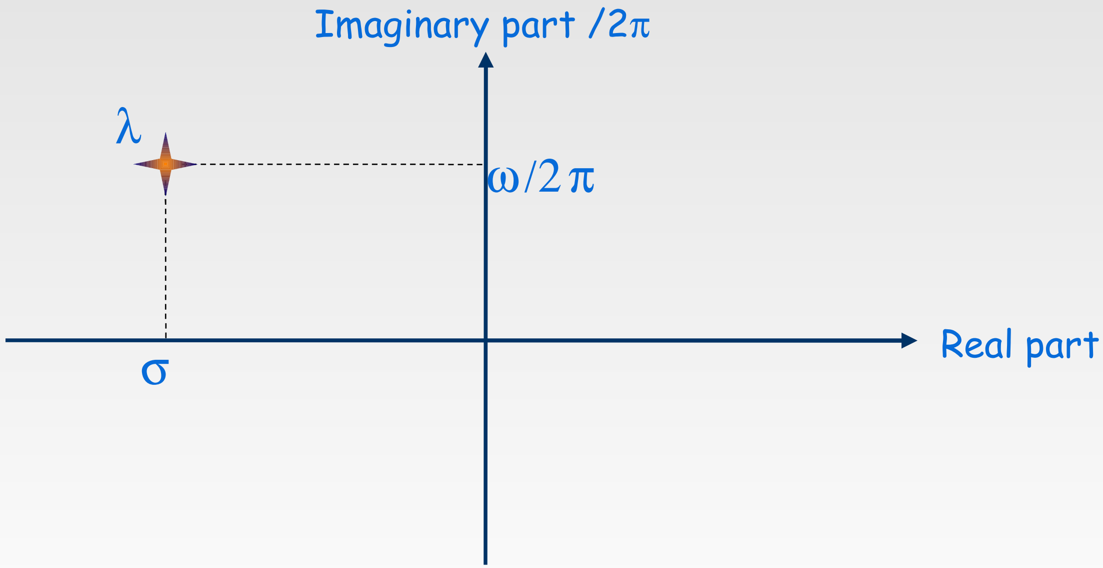
Frequencies and damping factor

- The damping factor ζ is a non-dimensional coefficient
- A critically damped mode, $\zeta = 1$, is non-oscillating, and returns slowly to steady state
- Under-damped ($\zeta < 1$) and over-damped ($\zeta > 1$) modes return to steady state slower than a critically damped mode
- The "natural frequency" is the frequency of the response on that specific mode
- The "undamped natural frequency" is a virtual value, if the mode was not damped
- For very low damping, i.e. $\zeta \ll 1$, the natural frequency is close to the undamped natural frequency

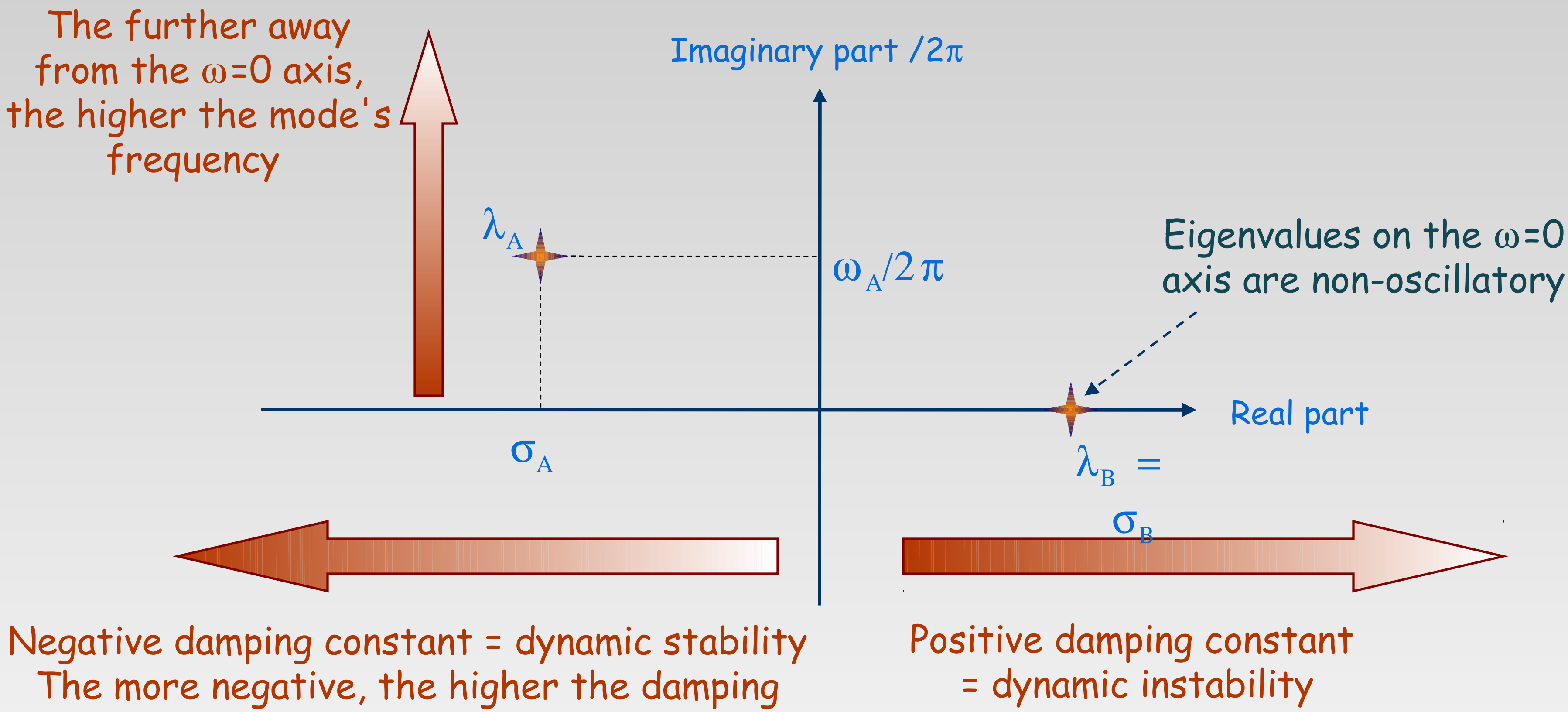


The root locus graph

- This graphic view provides a visual interpretation of the frequency and damping of a mode with eigenvalue $\lambda = \sigma_1 + i\omega_N$
- The time response of a mode component such as u, w, or q, is $f(t) = k.e^{\lambda t} = ke^{(\sigma_1 + i\omega_N)t}$
- ω_N is the natural circular frequency and $\omega_N/2\pi$ is the mode's natural frequency
- $\omega_1 = \sqrt{\sigma_1{}^2 + \omega_N{}^2}$ is the undamped natural circular frequency
- σ_1 is the damping constant and is related to the damping ratio by $\sigma_1 = -\omega_1\zeta$
- The eigenvalue is plotted in the $(\sigma_1,\ \omega_N/2\pi)$ axes, i.e. the root locus graph



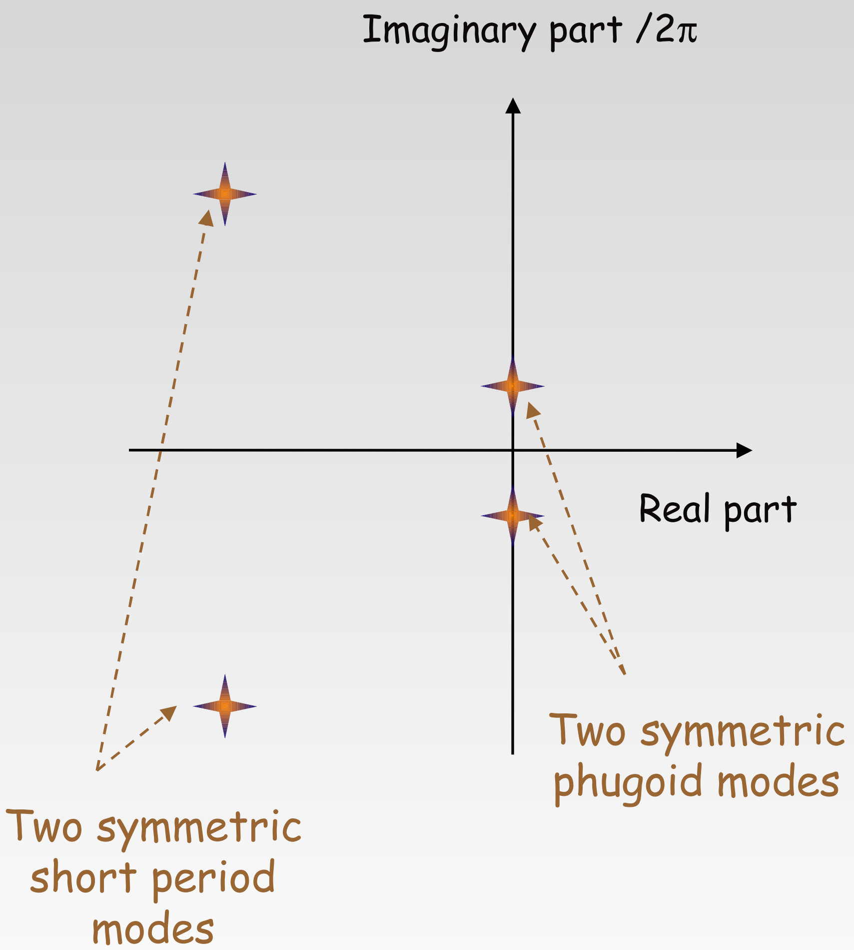
The root locus interpretation



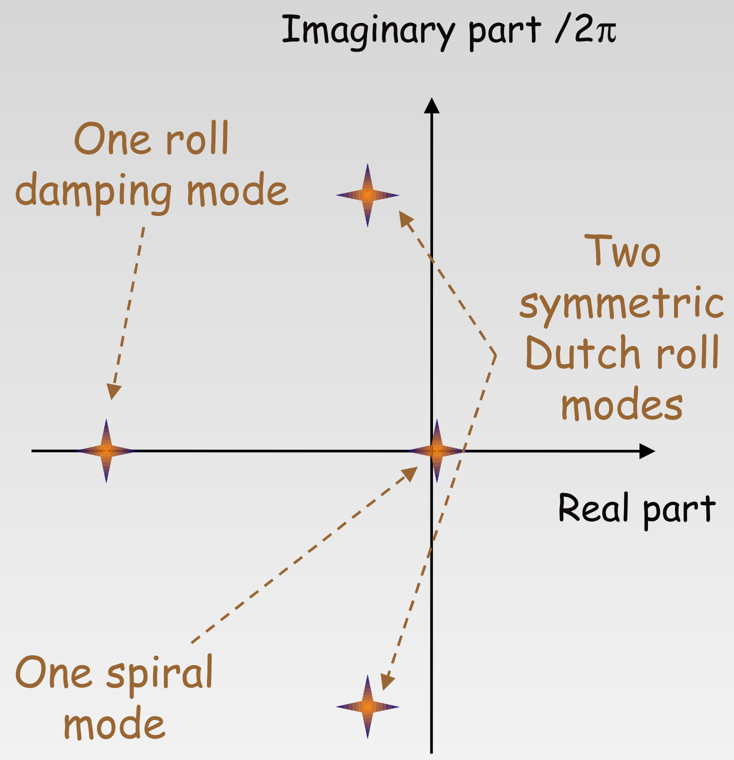
- λ_A corresponds to a damped oscillatory mode
- λ_B corresponds to an un-damped, non-oscillatory mode

The typical root locus graphs

Longitudinal

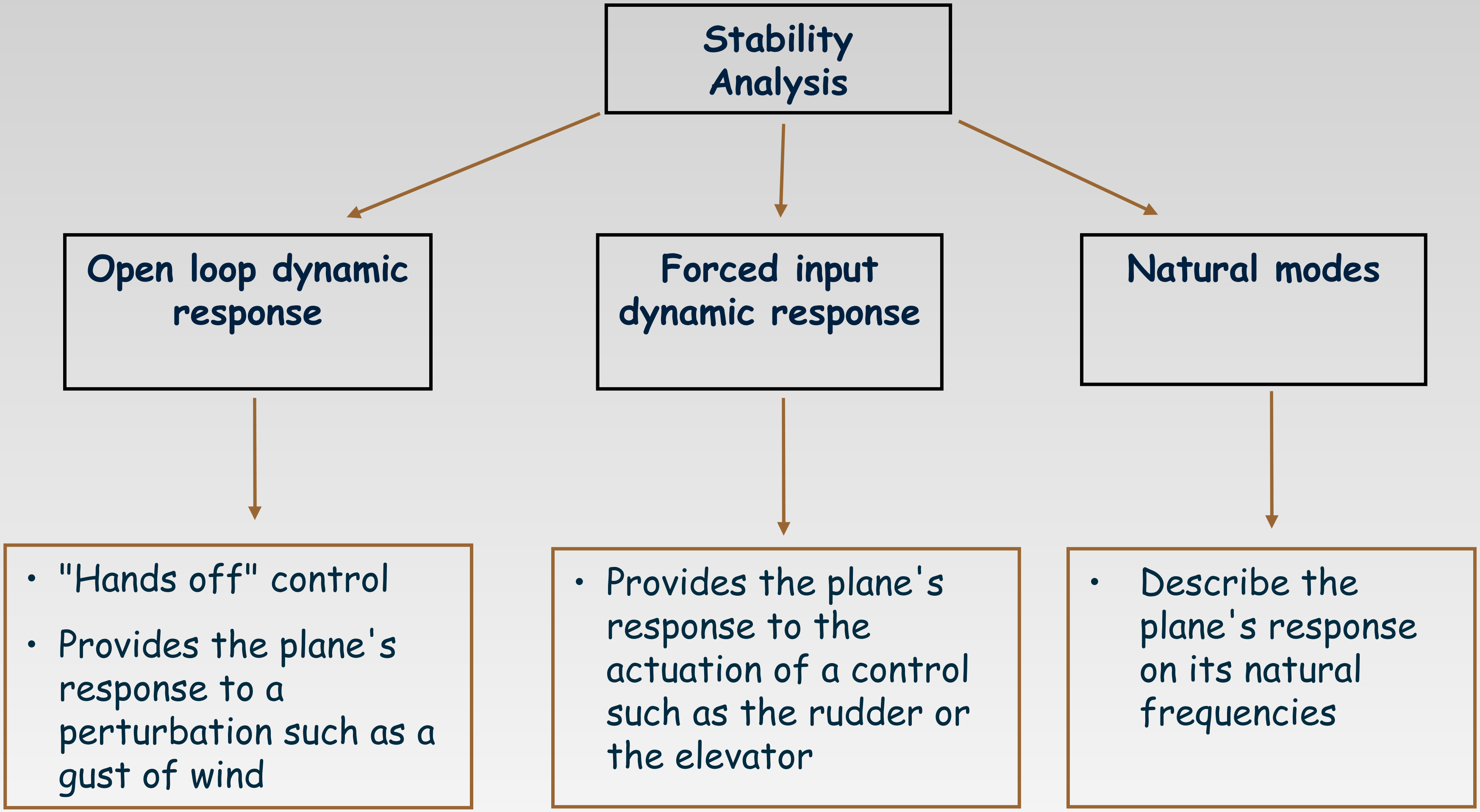


Lateral



Stability analysis in XFLR5

One analysis, three output



Pre-requisites for the analysis

- The stability and control behavior analysis requires that the inertia properties have been defined
- The evaluation of the inertia requires a full 3D CAD program
- Failing that, the inertia can be evaluated approximately in XFLR5 by providing
  - The mass of each wing and of the fuselage structure
  - The mass and location of such objects as nose lead, battery, receiver, servo-actuators, etc.
- XFLR5 will evaluate roughly the inertia based on these masses and on the geometry
- Once the data has been filled in, it is important to check that the total mass and CoG position are correct

Description of the steps of the analysis

Definition of geometry, mass and inertia

↓

Definition of the analysis/polar

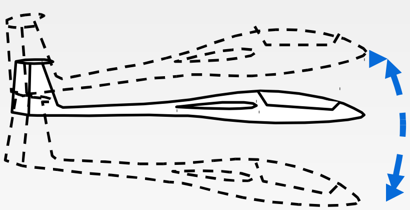
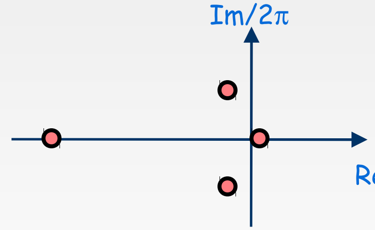
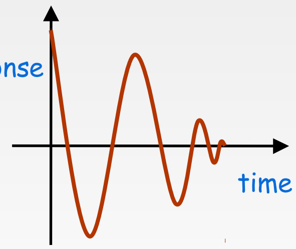
↓

Analysis

↓

Post-Processing

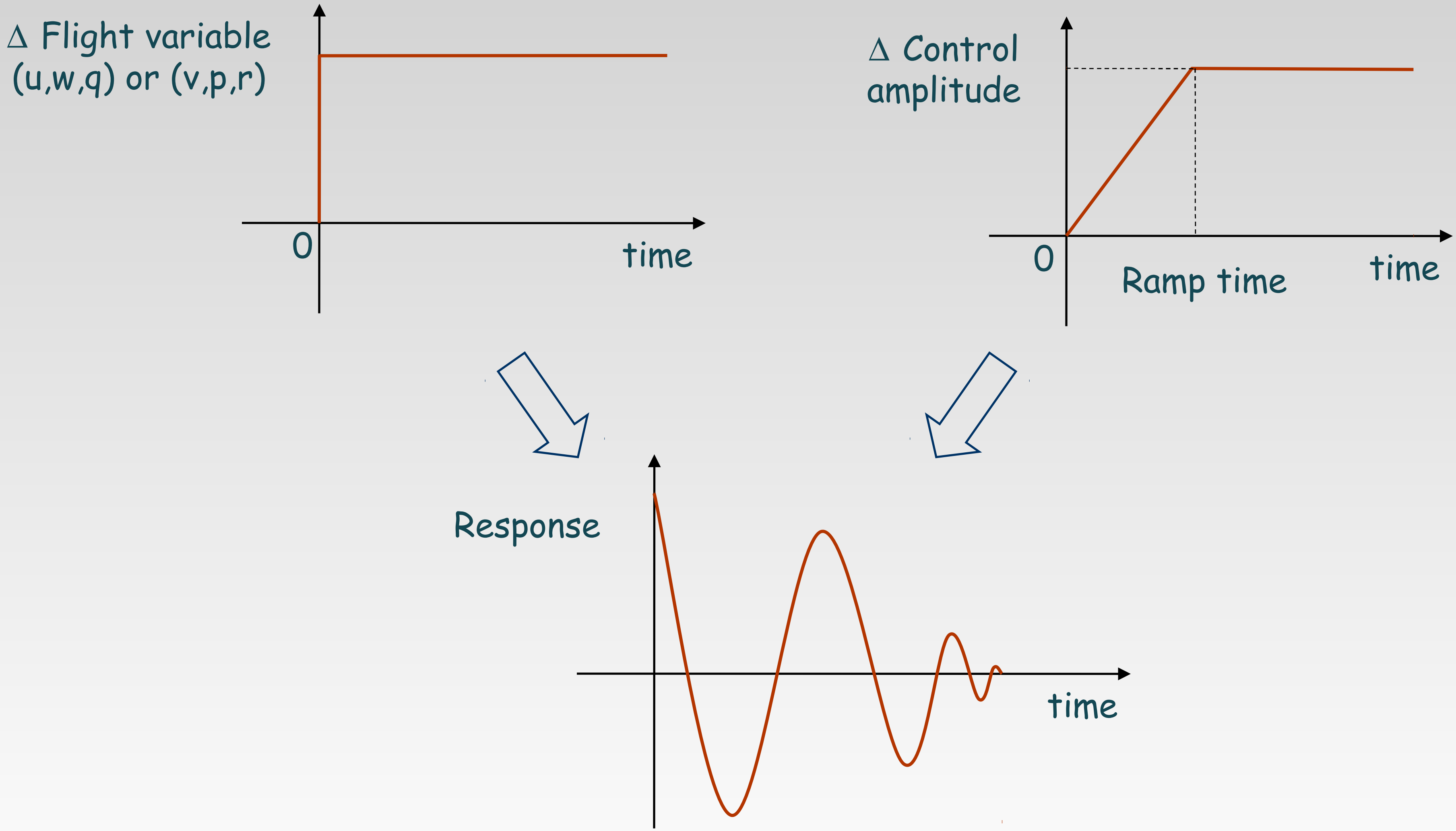
↓

3D-eigenmodes	Root locus graph	Time response
		

The time response view : two type of input

① Perturbation

② Control actuation



The 3D mode animation

- The best way to identify and understand a mode shape ?
- Note :
  - The apparent amplitude of the mode in the animation has no physical significance.
  - A specific mode is never excited alone in flight – the response is always a combination of modes.

Example of Longitudinal Dynamics analysis

Second approximation for the Short Period Mode

- **Taking into account the dependency to the vertical velocity leads to a more complicated expression**

$$t^* = \frac{MAC}{2u_0} \qquad \hat{I}_y = \frac{8I_y}{\rho.S.MAC^3} \qquad \mu = \frac{2m}{\rho.S.MAC}$$

u_0 = horizontal speed

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \qquad C_{z_\alpha} = \frac{\partial C_z}{\partial \alpha}$$

$C_{m\alpha}$ and $C_{z\alpha}$ are the slopes of the curves Cm = f(α) and Cz = f(α). The slopes can be measured on the polar graphs in XFLR5

$$B = \frac{C_{z_\alpha}}{2t^*\mu} \qquad C = -\frac{C_{m_\alpha}}{t^{*2}\hat{I}_y}$$

$$F_2 = \frac{1}{2\pi}\sqrt{-B^2 + 4C}$$

Despite their complicated appearance, these formula can be implemented in a spreadsheet, with all the input values provided by XFLR5

Lanchester's approximation for the Phugoid

- **The phugoid's frequency is deduced from the balance of kinetic and potential energies, and is calculated with a very simple formula**

$$F_{ph} = \frac{1}{\pi\sqrt{2}}\frac{g}{u_0}$$

g is the gravitational constant, i.e. g = 9.81 m/s
u_0 is the plane's speed

Numerical example – from a personal model sailplane

- Plane and flight Data

MAC =	0.1520	m
Mass =	0.5250	kg
Iyy =	0.0346	kg.m²
S =	0.2070	m²
ρ =	1.225	kg/m3

u0 =	16.20	m/s
α =	1.05	°
q =	160.74	Pa

Cx =	0.0114	
Cz =	0.1540	
dCm/dα =	-1.9099	
dCz/dα =	-5.3925	

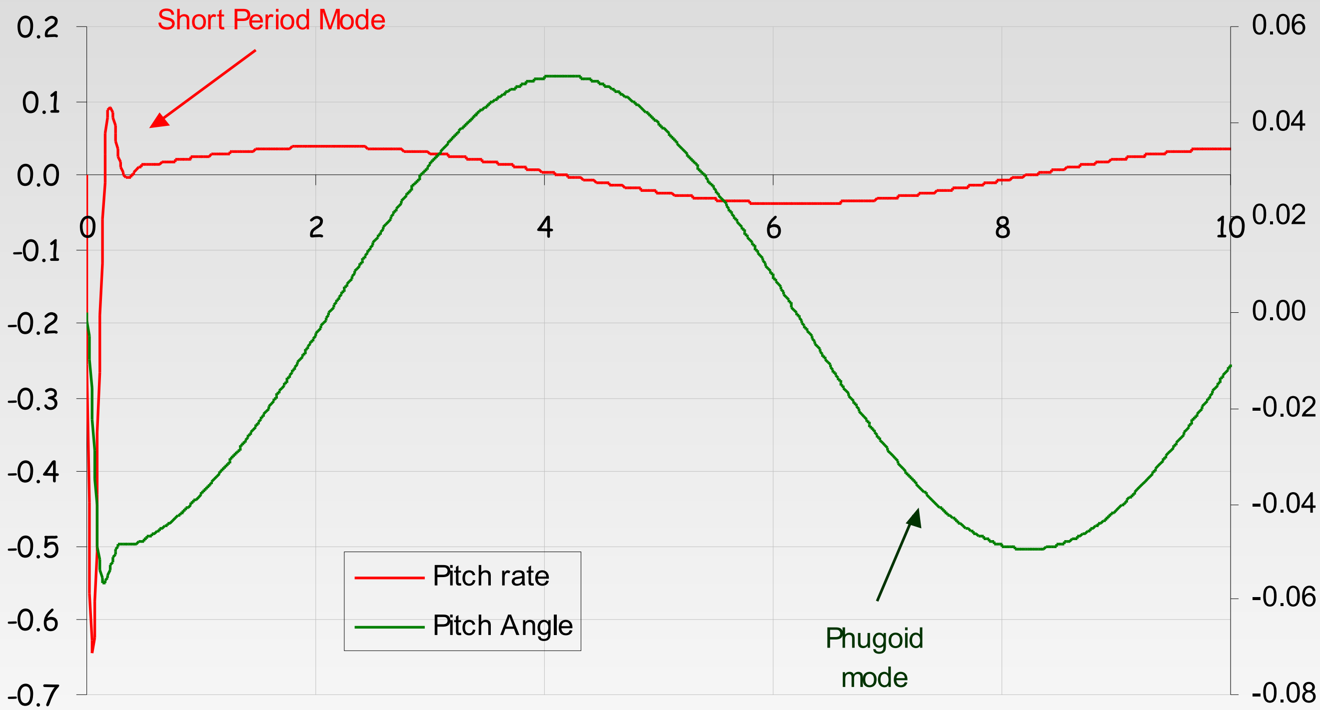
- Results

	Short Period			Phugoid	
	F1	F2	XFLR5 v6	Fph	XFLR5 v6
Frequency (Hz) =	4.45	4.12	3.86	0.136	0.122
Period (s) =	0.225	0.243	0.259	7.3	8.2

Graphic Analysis →

Time response

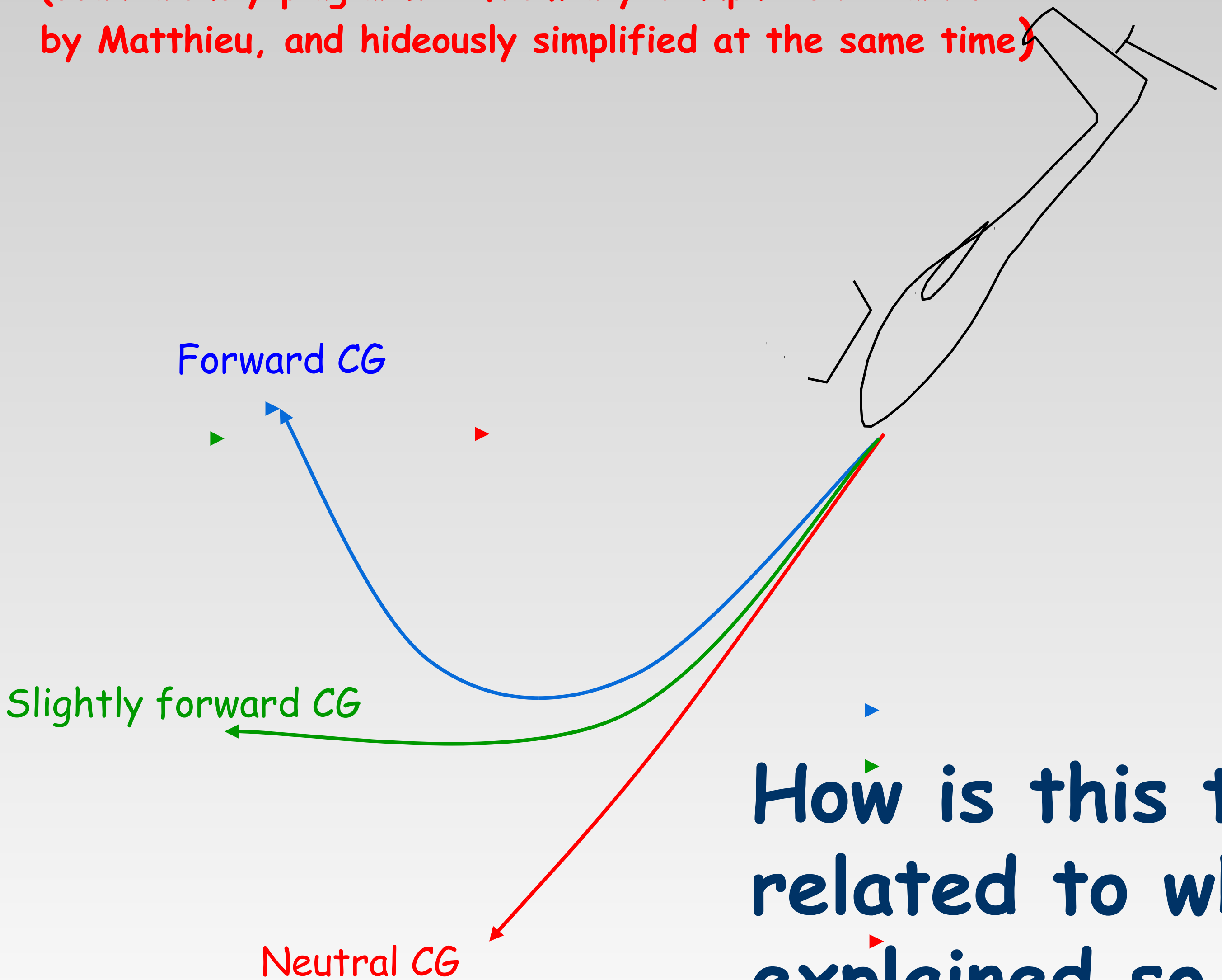
- There is factor 40x between the numerical frequencies of both modes, which means the plane should be more than stable
- A time response analysis confirms that the two modes do not interact



About the Dive Test

About the dive test

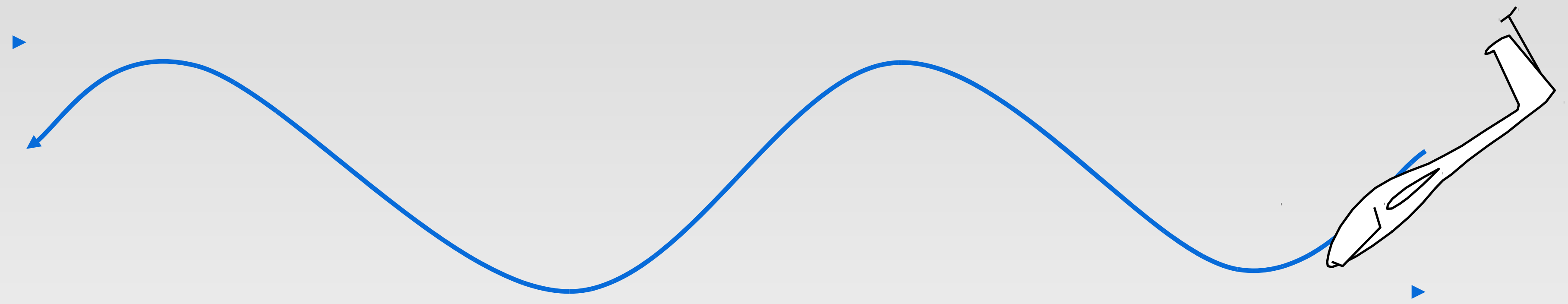
(scandalously plagiarized from a yet unpublished article by Matthieu, and hideously simplified at the same time)



How is this test related to what's been explained so far?

Forward CG

- If the CG is positioned forward, the plane will enter the phugoid mode

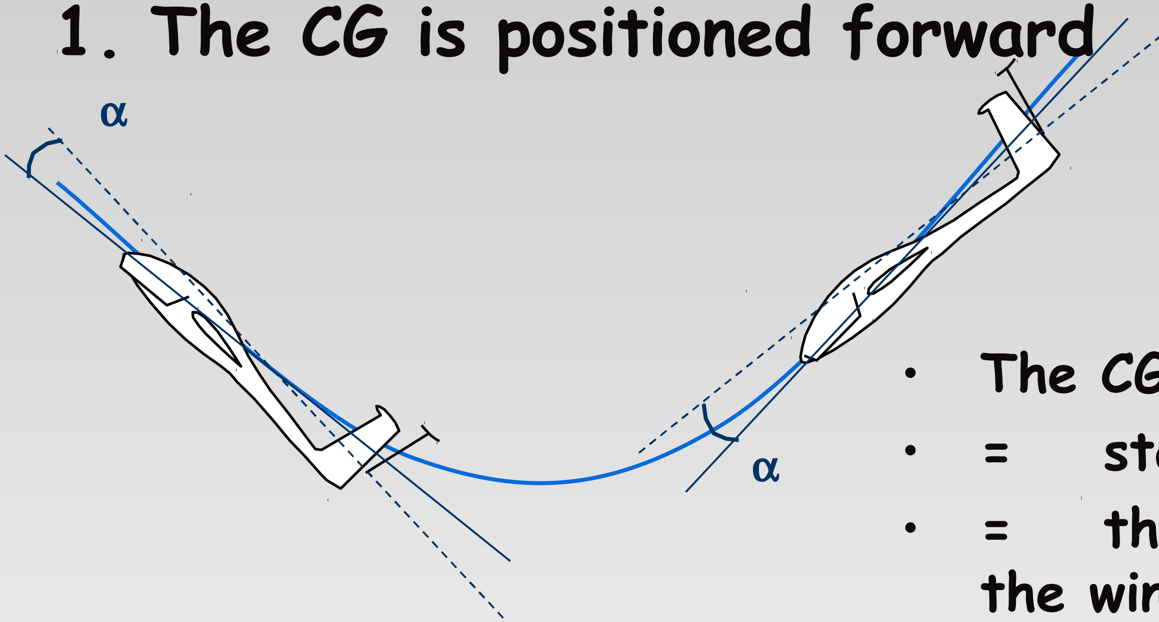


Stick to the phugoid

- **As the plane moves along the phugoid, the apparent wind changes direction**
- **From the plane's point of view, it's a perturbation**
- **The plane can react and reorient itself along the trajectory direction, providing**
  - That the slope of the curve Cm = f(α) is stiff enough
  - That it doesn't have too much pitching inertia

Summarizing :

1. The CG is positioned forward



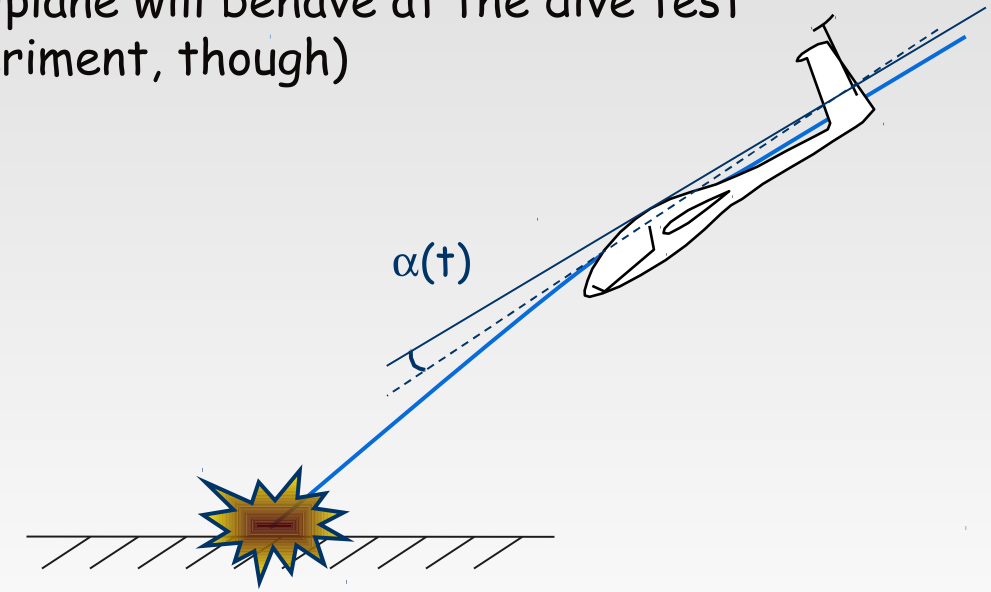
- The CG is positioned forward
- = stability
- = the wind vane which follows the wind gusts

- The two modes are un-coupled
- The relative wind changes direction along the phugoid…
- … but the plane maintains a constant incidence along the phugoid, just as the chariot remains tangent to the slope
- The sailplane enters the phugoid mode

2. The CG is positioned aft

•Remember that backward CG = instability = the wind vane which amplifies wind gusts

- The two modes are coupled
- The incidence oscillation $\alpha(t)$ amplifies the phugoid,
- The lift coefficient is not constant during the phugoid
- The former loop doesn't work any more
- The phugoid mode disappears
- No guessing how the sailplane will behave at the dive test
  (It's fairly easy to experiment, though)



That's all for now

Good design and nice flights ☺

Needless to say, this presentation owes a lot to Matthieu Scherrer ; thanks Matt !