Overview of the theoretical background in XFLR5

Watch on YouTube:
Part I: Theoretical overview
Part II: The inviscid problem
Part III: The viscous flow
The million dollar problem: solving the Navier-Stokes Equations

(1) \[ \frac{\partial}{\partial t} u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x, t) \quad (x \in \mathbb{R}^n, t \geq 0), \]

(2) \[ \text{div } u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \quad (x \in \mathbb{R}^n, t \geq 0) \]

with initial conditions

(3) \[ u(x, 0) = u^0(x) \quad (x \in \mathbb{R}^n). \]
The million dollar problem: solving the Navier-Stokes Equations

\[ \frac{\partial}{\partial t} u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x, t) \quad (x \in \mathbb{R}^n, t \geq 0), \]

(2) \[ \text{div} \ u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \quad (x \in \mathbb{R}^n, t \geq 0) \]

with initial conditions

(3) \[ u(x, 0) = u^0(x) \quad (x \in \mathbb{R}^n). \]

The Navier-Stokes equations are to fluid dynamics what Maxwell’s equations are to electromagnetism (more on that later)
The million dollar problem: solving the Navier-Stokes Equations

\[ \frac{\partial}{\partial t} u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t) \quad (x \in \mathbb{R}^n, t \geq 0), \]

\[ \text{div} \ u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \quad (x \in \mathbb{R}^n, t \geq 0) \]

with initial conditions

\[ u(x,0) = u^0(x) \quad (x \in \mathbb{R}^n). \]

“Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.”

One of the seven millennium prize problems published by The Clay Mathematics Institute
The million dollar problem: solving the Navier-Stokes Equations

\[
\frac{\partial}{\partial t} u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t) \quad (x \in \mathbb{R}^n, t \geq 0),
\]

\[
\text{div} \, u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \quad (x \in \mathbb{R}^n, t \geq 0)
\]

with initial conditions

\[
u(x,0) = u^0(x) \quad (x \in \mathbb{R}^n).
\]

Computational Fluid Dynamics is all about solving numerically the Navier-Stokes equations.
Navier-Stokes equations
Navier-Stokes equations

Euler’s equations

\[
\frac{Du}{Dt} = -\nabla w + g \\
\nabla \cdot u = 0
\]
Navier-Stokes equations

Inviscid fluid

Euler's equations

Irrotational flow

\[ \vec{V} = \nabla \phi \]

Potential flow
Navier-Stokes equations

Euler’s equations

Inviscid fluid

irrotational flow

$\vec{V} = \nabla \phi$

Potential flow

Time independent, incompressible flow

Laplace’s equation

$\Delta \phi = 0$

The same equation governs electrostatics

Fluid potential ↔ Electrostatic potential

Fluid velocity field ↔ Electric field
Navier-Stokes equations

Euler’s equations

Inviscid fluid

Irrotational flow

\[ \vec{V} = \nabla \phi \]

Potential flow

Time independent, incompressible flow

Laplace’s equation

\[ \Delta \phi = 0 \]

2d, 3d

From Richard Feynman Lectures:

“When we drop the viscosity term, we will be making an approximation which describes some ideal stuff rather than real water. John von Neumann was well aware of the tremendous difference [...], [...] the main interest was in solving beautiful mathematical problems with this approximation which had almost nothing to do with real fluids. He characterized the theorist who made such analyses as a man who studied “dry water.” [...]. We are postponing a discussion of real water to the next chapter.”

http://www.feynmanlectures.caltech.edu/II_40.html
Navier-Stokes equations

Euler’s equations

Reynolds equations

Inviscid fluid

Irrotational flow

\[ \vec{V} = \nabla \phi \]

Potential flow

Time independent, incompressible flow

Laplace’s equation

\[ \Delta \phi = 0 \]

2d, 3d

CFD « RANS »

Reynolds Averaged Navier-Stokes solvers

Time averaged turbulence
In aerodynamics, the main effect of viscosity is to create a thin Boundary Layer (BL) on all lifting and non-lifting surfaces.
Navier-Stokes equations

Inviscid fluid

Euler’s equations

irrotational flow
\[ \vec{V} = \nabla \phi \]

Potential flow

Time independent, incompressible flow

Laplace’s equation
\[ \Delta \phi = 0 \]

Reynolds equations

Time averaged turbulence

3d Boundary Layer eq.

Viscosity models, uniform pressure in BL thickness, Prandlt mixing length hypothesis.

CFD « RANS » Reynolds Averaged Navier-stokes solvers

2d, 3d
Navier-Stokes equations

Inviscid fluid

Euler’s equations

\( \vec{V} = \nabla \phi \)

Potential flow

Irrotational flow

Laplace’s equation

\( \Delta \phi = 0 \)

Time independent, incompressible flow

Reynolds equations

Time averaged turbulence

3d Boundary Layer eq.

2d BL equations

2d, 3d

CFD « RANS »
Reynolds Averaged Navier-stokes solvers

Viscosity models, uniform pressure in BL thickness, Prandlt mixing length hypothesis.
Navier-Stokes equations

- Euler’s equations
  - Irrotational flow: $\vec{V} = \nabla \phi$
  - Potential flow
  - Laplace’s equation: $\Delta \phi = 0$

- Reynolds equations
  - Time averaged turbulence
  - 3d Boundary Layer equation
  - 2d BL equations
    - 1d BL Integral equations
    - 2d BL differential equations

- Time independent, incompressible flow

CFD « RANS »
Reynolds Averaged Navier-stokes solvers

Viscosity models, uniform pressure in BL thickness, Prandtl mixing length hypothesis.

mixed empirical + theoretical turbulence and transition models

xflr5
Subsonic Airfoil Development System
Why does a plane fly:
the inviscid potential flow