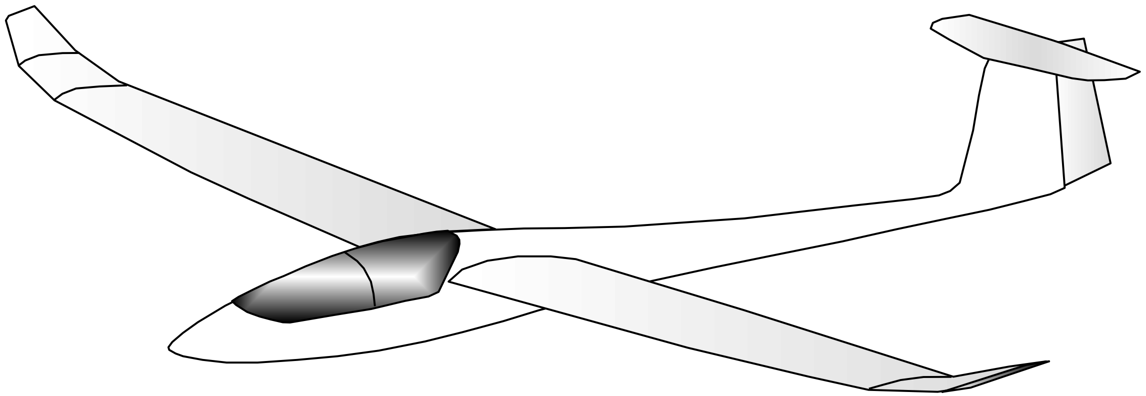

Overview of the theoretical background in xflr5

Watch on YouTube:

Part I: Theoretical overview

Part II: The inviscid problem

Part III: The viscous flow

The million dollar problem: solving the Navier-Stokes Equations

$$(1) \qquad \frac{\partial}{\partial t} u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t) \qquad (x \in \mathbb{R}^n, t \geq 0),$$

$$(2) \qquad \operatorname{div} u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \qquad (x \in \mathbb{R}^n, t \geq 0)$$

with initial conditions

$$(3) \qquad u(x,0) = u^\circ(x) \qquad (x \in \mathbb{R}^n).$$

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The Navier-Stokes equations are to fluid dynamics what Maxwell's equations are to electromagnetism

(more on that later)

The million dollar problem: solving the Navier-Stokes Equations

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with initial conditions

$$\text{(3)} \qquad u(x,0) = u^{\circ}(x) \qquad (x \in \mathbb{R}^n).$$

"Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations."

One of the seven millennium prize problems published by The Clay Mathematics Institute

The million dollar problem: solving the Navier-Stokes Equations

$$(1) \qquad \frac{\partial}{\partial t}u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t) \qquad (x \in \mathbb{R}^n, t \geq 0),$$

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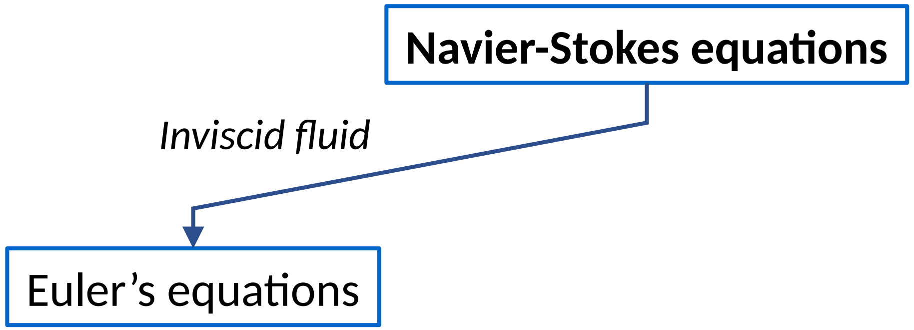
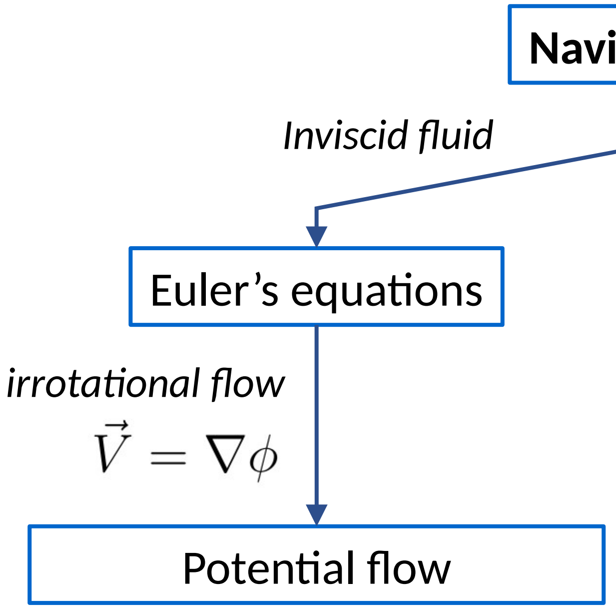
Computational Fluid Dynamics is all about solving numerically the Navier-Stokes equations

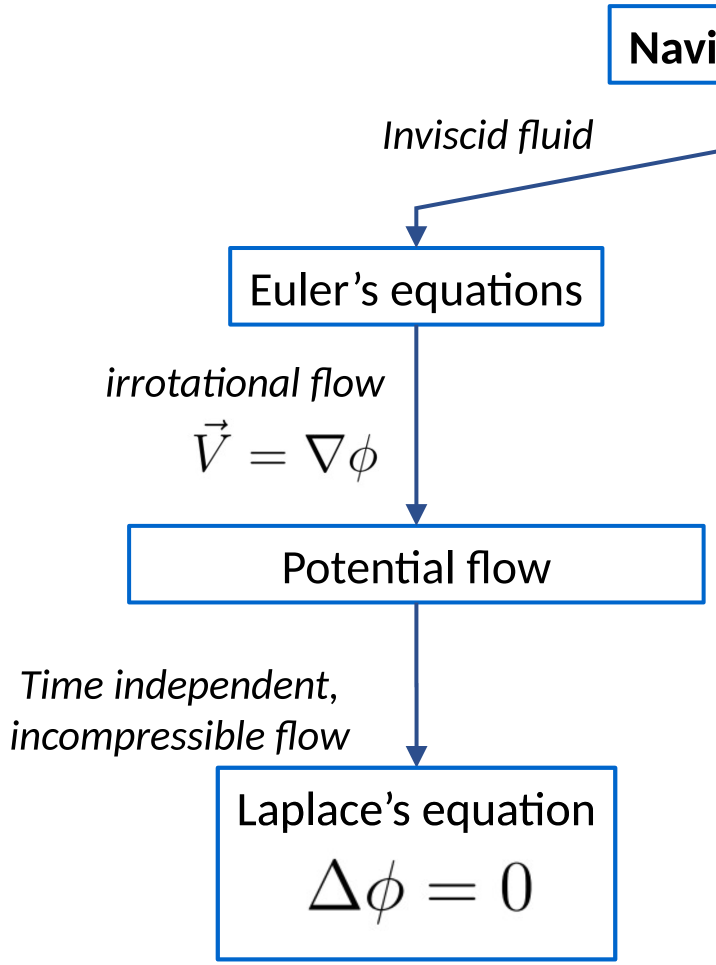
Navier-Stokes equations



$$\frac{Du}{Dt} = -\nabla w + g$$

$$\nabla . u = 0$$

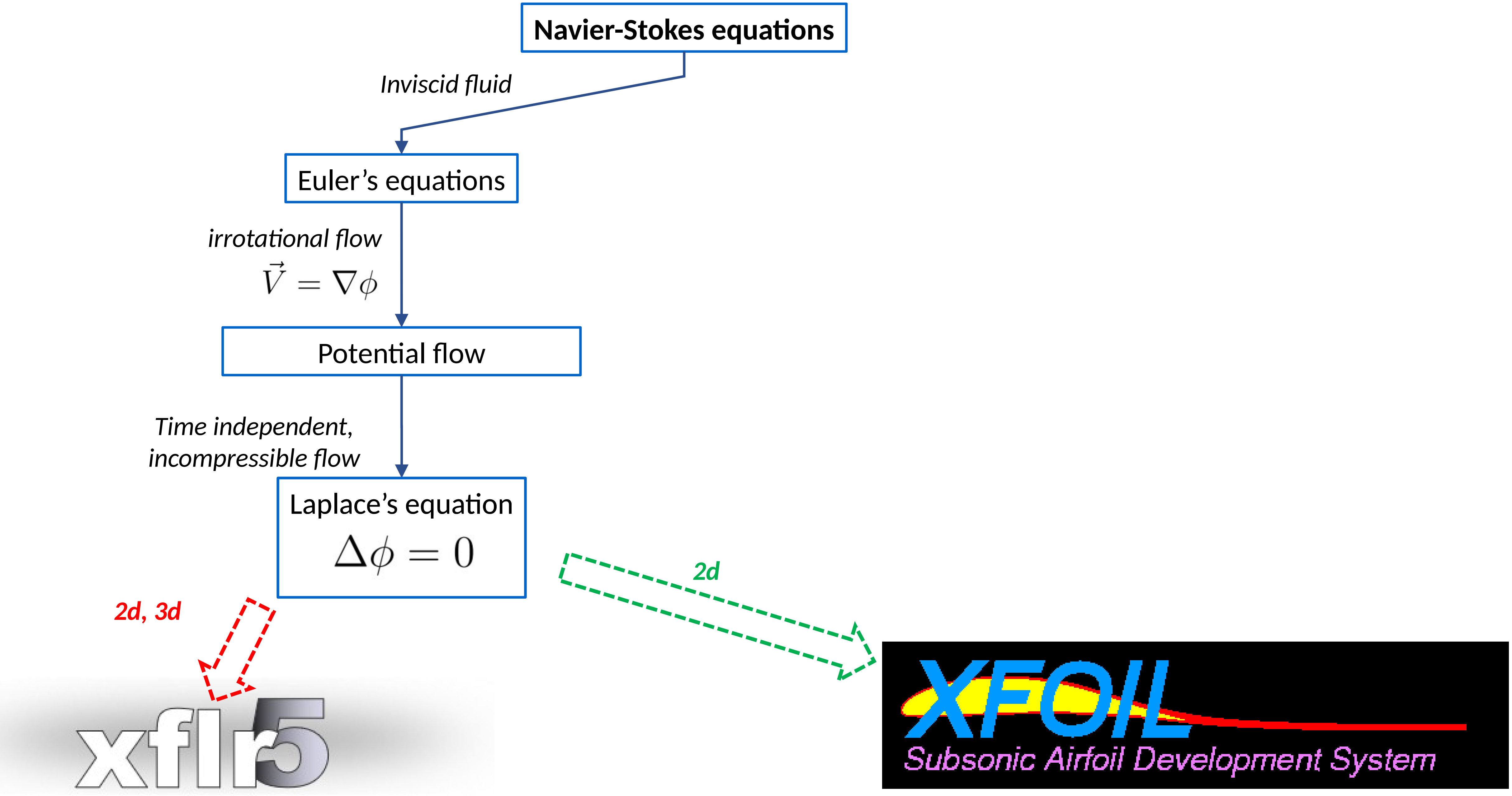

Navier-Stokes equations
Inviscid fluid
Euler's equations
irrotational flow
$\vec{V} = \nabla\phi$
Potential flow



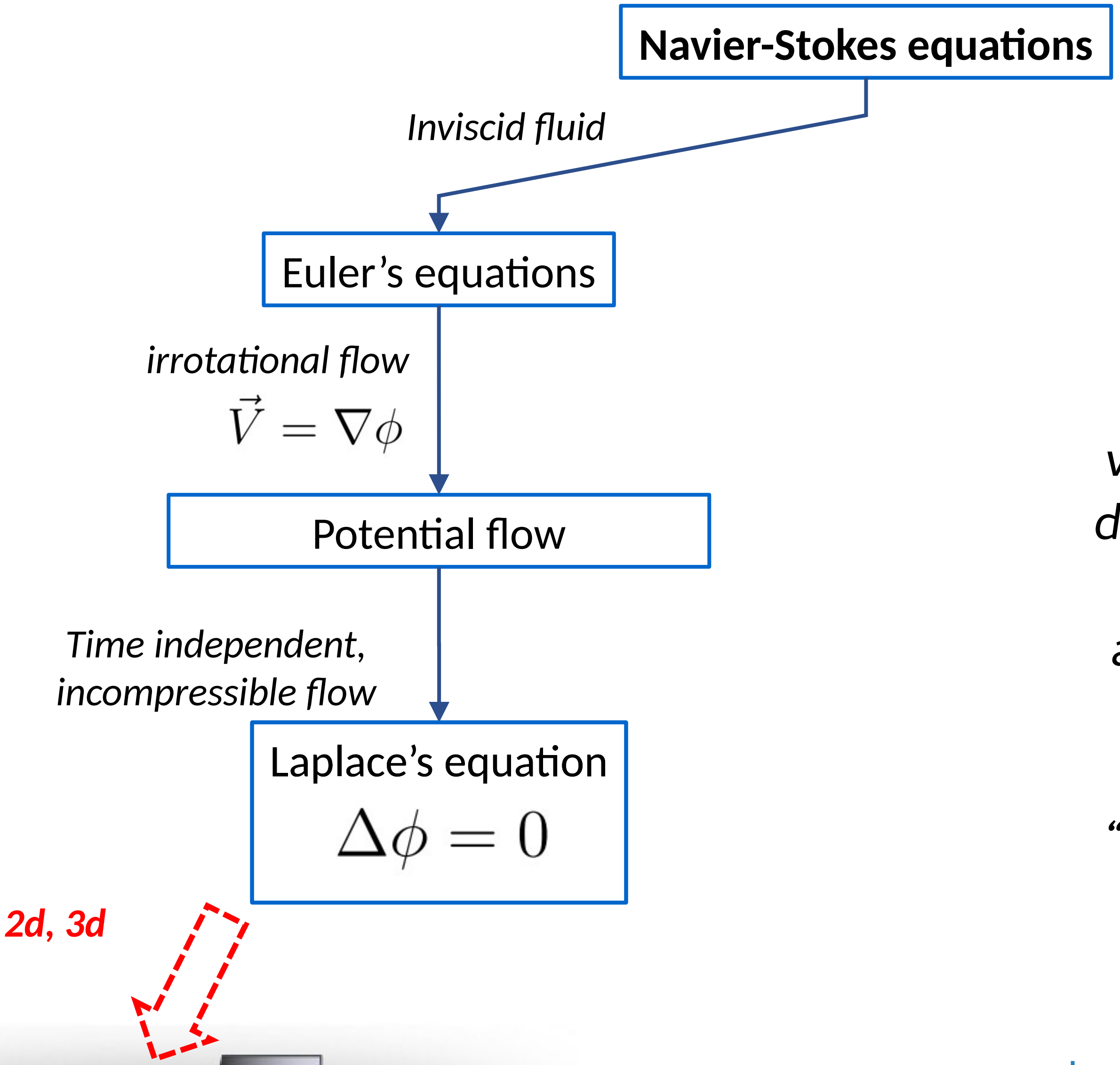
The same equation governs electrostatics

Fluid potential ⟺ **Electrostatic potential**

Fluid velocity field ⟺ **Electric field**



Navier-Stokes equations
Inviscid fluid
Euler's equations
irrotational flow
$\vec{V} = \nabla\phi$
Potential flow
Time independent,
incompressible flow
Laplace's equation
$\Delta\phi = 0$
2d
2d, 3d
xflr5
XFOIL
Subsonic Airfoil Development System



xflr5

From Richard Feynman Lectures:

"When we drop the viscosity term, we will be making an approximation which describes ***some ideal stuff rather than real water****. John von Neumann was well aware of the tremendous difference [...], [...] the main interest was in solving beautiful mathematical problems with this approximation* ***which had almost nothing to do with real fluids****. He characterized the theorist who made such analyses as a man who studied "dry water." [...]. We are postponing a discussion of* ***real water*** *to the next chapter."*

http://www.feynmanlectures.caltech.edu/II_40.html

Navier-Stokes equations

Inviscid fluid → Euler's equations

irrotational flow

$$\vec{V} = \nabla\phi$$

→ Potential flow

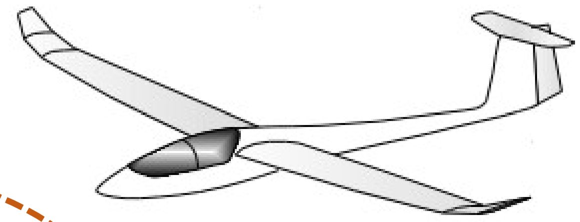
Time independent, incompressible flow → Laplace's equation

$$\Delta\phi = 0$$

2d, 3d → xflr5

Time averaged turbulence → Reynolds equations

CFD « RANS »
Reynolds Averaged
Navier-stokes solvers



Navier-Stokes equations

Inviscid fluid → **Euler's equations**

irrotational flow

$\vec{V} = \nabla\phi$

→ **Potential flow**

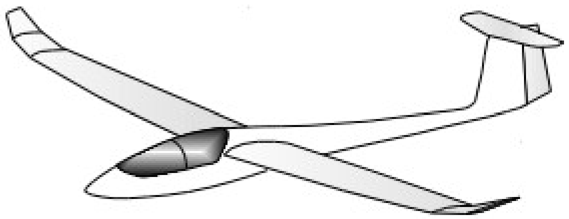
Time independent, incompressible flow → **Laplace's equation**

$$\Delta\phi = 0$$

2d, 3d

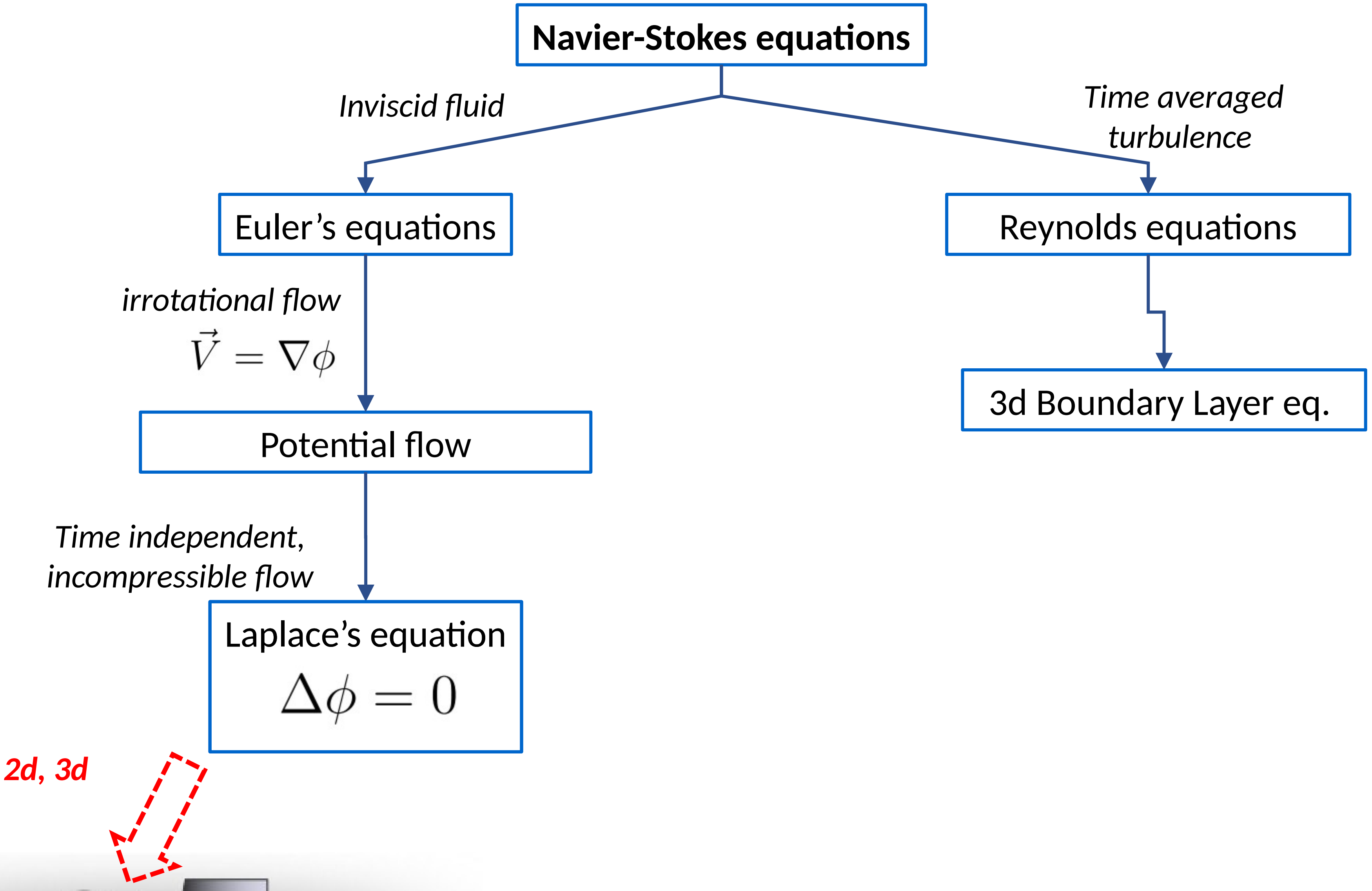
xflr5

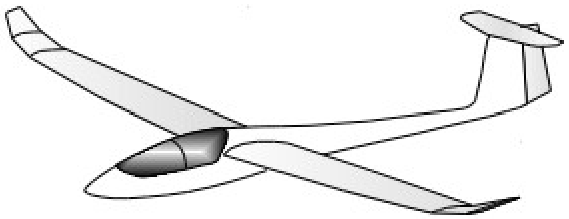
Time averaged turbulence → **Reynolds equations** → **3d Boundary Layer eq.**



CFD « RANS »
Reynolds Averaged
Navier-stokes solvers

In aerodynamics, the main effect of viscosity is to create a thin Boundary Layer (BL) on all lifting and non-lifting surfaces





CFD « RANS »
Reynolds Averaged
Navier-stokes solvers

Viscosity models, uniform pressure in BL thickness, Prandlt mixing length hypothesis.

Navier-Stokes equations

Inviscid fluid

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xflr5

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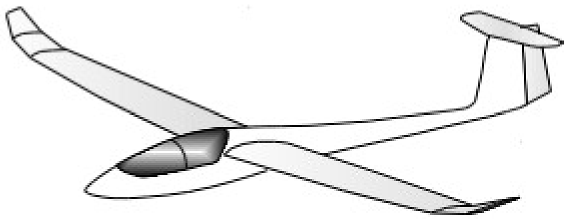
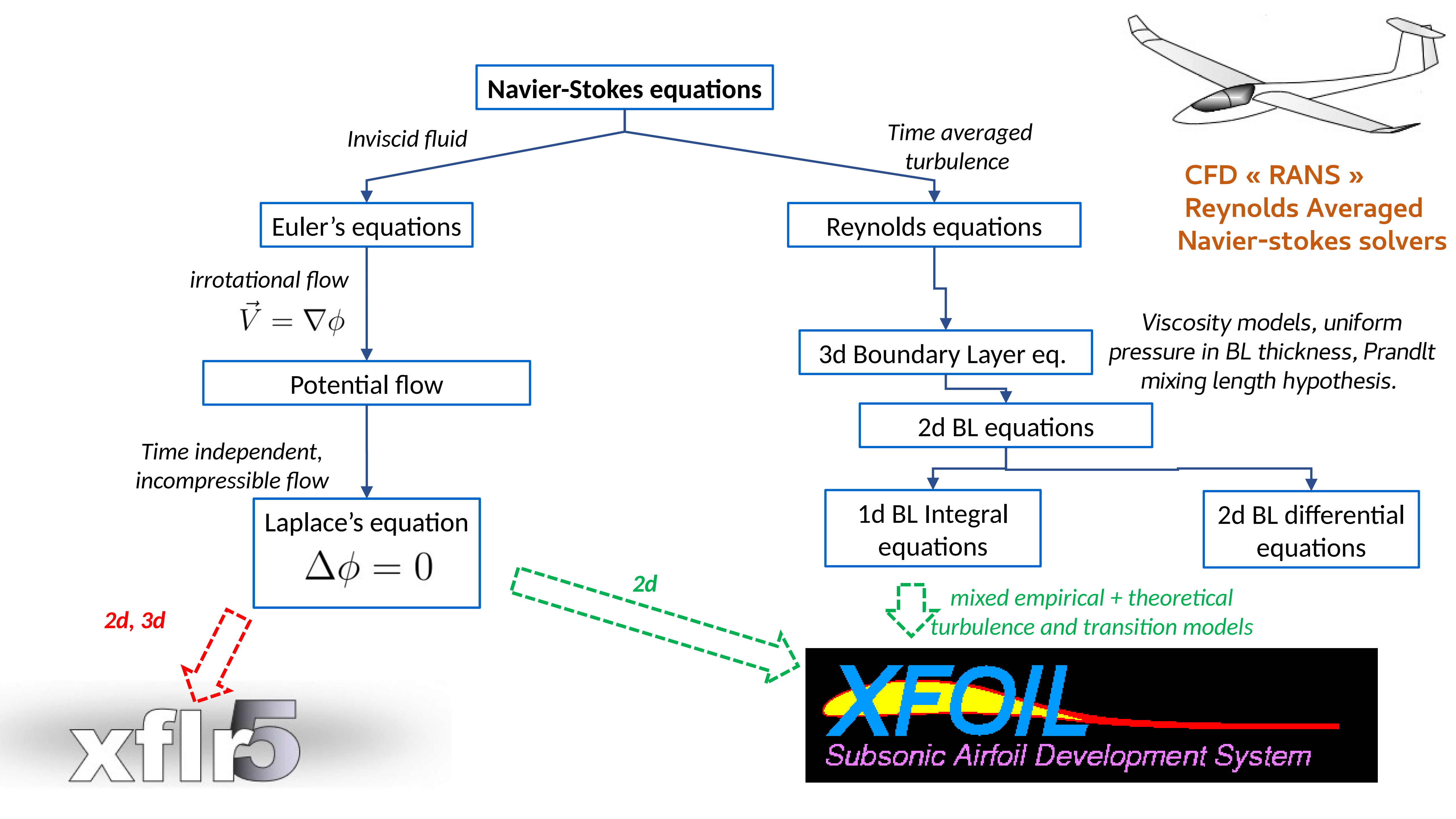
2d BL equations

1d BL Integral equations

2d BL differential equations



CFD « RANS »
Reynolds Averaged Navier-stokes solvers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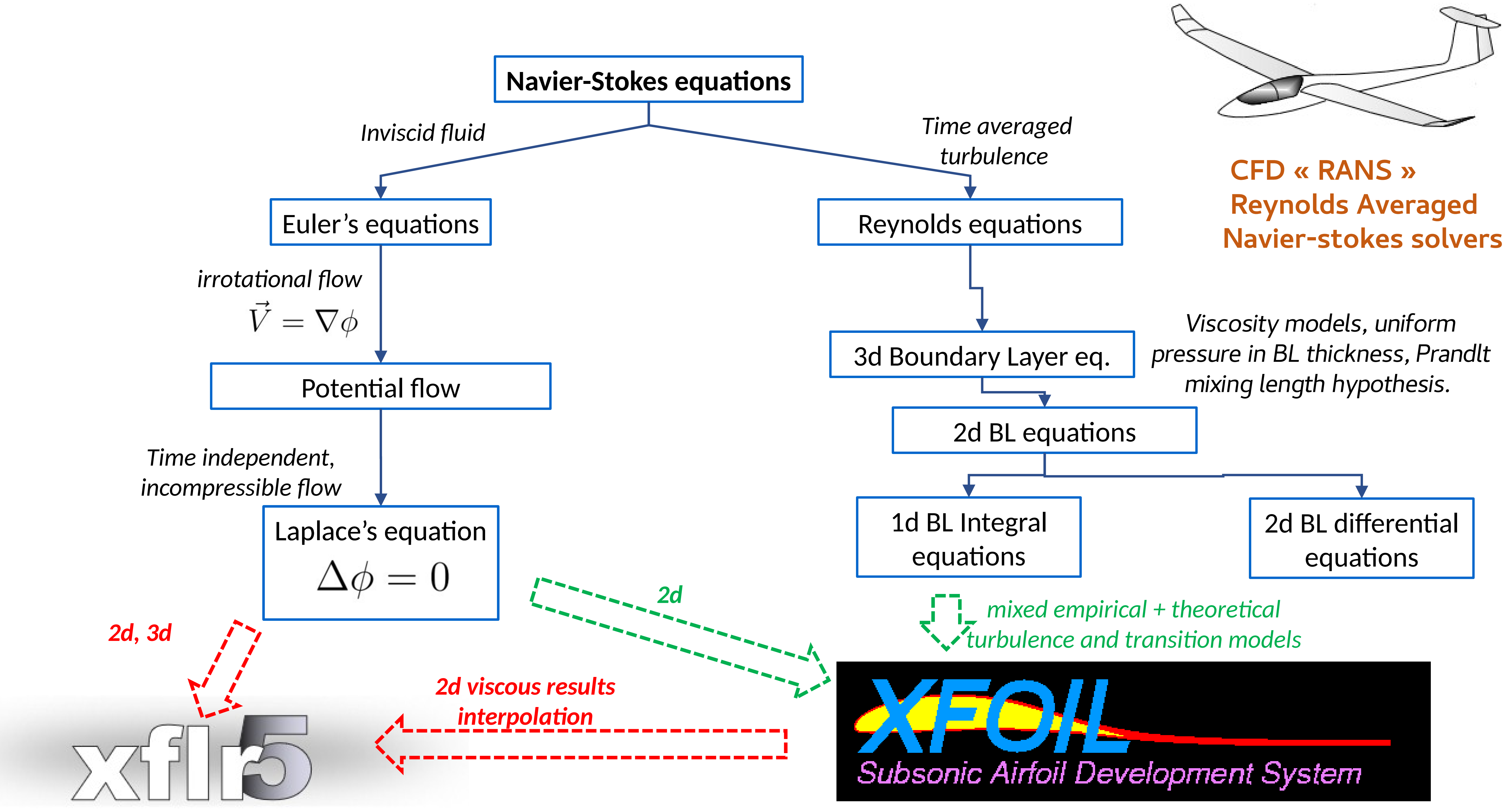

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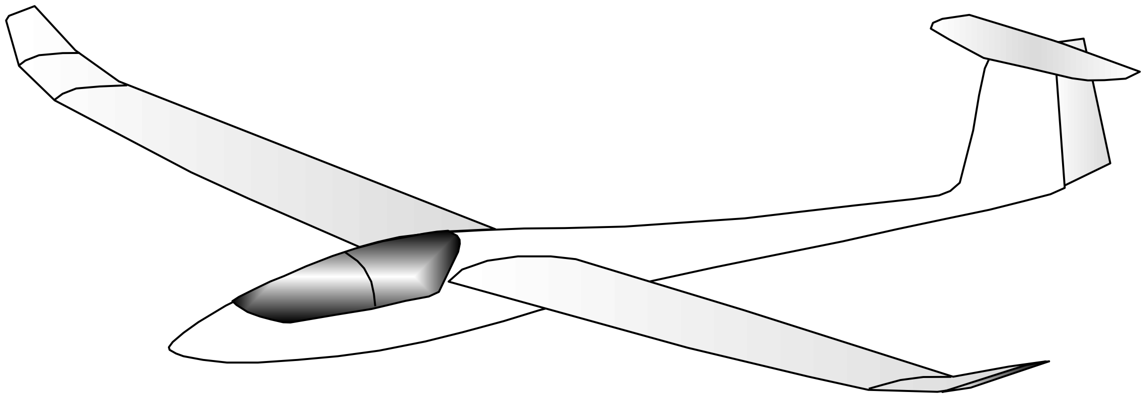

- up next -

Why does a plane fly:
the inviscid potential flow